MATH 3P85FIRST MIDTERMOctober 5, 2015Full credit given for (the best) 3 correct and complete answers.Answers must be given to at least 4 significant digits.Open-book exam.Duration: 1 hour

1. Consider a random independent sample of size 105 from the following distribution:

X =	-2	0	1	3	7
Pr	0.17	0.28	0.36	0.15	0.04

Using the Normal approximation (a.k.a. CLT) with continuity correction, estimate the $Pr(\bar{X} < 1)$, where \bar{X} is the corresponding sample mean; compare with the exact answer.

2. Knowing that A, B, C and D are *independent* and Pr(A) = 0.6, Pr(B) = 0.7, Pr(C) = 0.3 and Pr(D) = 0.5, compute the

 $\Pr[(\overline{A} \cup B \cup \overline{C} \cup D) \cap (A \cup \overline{B} \cup C \cup \overline{D}) \cap (\overline{A} \cup B \cup C \cup \overline{D})]$

Hint: find the complement first (a partial mark given for this alone).

- 3. A team will play a series of 10 independent games, each of which they can win, lose or tie with the probability of 45%, 50% and 5% respectively. They earn 3 points for each win, 1 point for a tie, and no points when losing. Compute the expected value of the number of points they earn playing this series and the corresponding standard deviation. Also, what is the probability that they will earn at least 15 points in total (hint: find the PGF of the number of points earned in a single game, raise it to the power of 10, expand, etc.).
- 4. The bivariate PDF of two random variables X and Y is given by

$$f(x,y) = \begin{cases} y \cdot e^{-x} & \text{when } 0 < y < x \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal PDF of each X and Y (do not forget to specify the corresponding support), and the conditional PDF of X given that Y = 1 (and its support). Are X and Y independent (substantiate your answer)?

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5. Using the distribution of the previous question, compute the correlation coefficient between X and Y, and also the $\Pr(X + Y \le 1)$.