MATH 3P85FIRST MIDTERMFeb. 14, 2017Full credit given for 3 (out of 5) correct answers.Quote all numbers in DECIMAL, to at least 4 significant digits.Enter all final answers (usually a single number) in the booklet.Email your Maple to jvrbik@brocku.ca (and keep a copy).Open-book exam.Duration: 1 hour

1. Consider playing a game with the following pay-off table, independently, 200 times:

X =	-2	0	4	10
Pr	0.56	0.28	0.14	0.02

Using the Normal approximation (aka CLT) with continuity correction, estimate the probability of losing more than \$50. Also, compute the exact answer.

2. Assuming A, B, C and D to be mutually *independent* events with Pr(A) = 0.62, Pr(B) = 0.71, Pr(C) = 0.38 and Pr(D) = 0.55, compute

 $\Pr[(\overline{A} \cap B \cap C \cap D) \cup (A \cap \overline{B} \cap C) \cup (C \cap \overline{D}) \cup B]$ 

3. A random variable X has the following pdf:

$$f(x) = \begin{cases} \frac{1}{2} + x & -\frac{1}{2} \le x < 0\\ \frac{3}{4} & 0 \le x < \frac{1}{2}\\ c(1-x) & \frac{1}{2} \le x < 1\\ 0 & \text{otherwise} \end{cases}$$

Find

- (a) the value of c,
- (b)  $\Pr(-0.1 < X < 0.6),$
- (c)  $\operatorname{Var}(X)$ .
- 4. Suppose 7 cards are randomly dealt from a regular deck (of 52 cards), and 10 points awarded for each ace and 3 points for each spade (13 points for the ace of spades) found in the resulting hand. Let X denote the number of points received. Compute

- (a) the mean and standard deviation of X,
- (b) Pr(X > 15). Hint: use the corresponding PGF.
- 5. Assuming that X and Y have the following joint pdf

$$f(x,y) = \begin{cases} c \cdot (1+x^2) \cdot \frac{\exp(-2x-y)}{\sqrt{y}} & \text{when } x > 0 & \text{and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

find

- (a) the value of c
- (b) and  $\Pr(3X + 2Y > 4)$ .
- (c) Are X and Y independent (substantiate your answer)?