## MATH 3P85Second midterm16 Nov. 2015Full credit given for 5 (out of 10) correct and complete answers.Open-book exam.Duration: 1 hour

- 1. Let X and Y be two independent RVs having the exponential distribution with the (different!) mean of 1 and 2 respectively. Find the PDF of:
  - (a)  $U \equiv X + Y$  (hint: use convolution),
  - (b)  $V \equiv \min(X, Y)$  and identify the resulting distribution. Hint: first find  $1 - F_V(v) = \Pr\{\min(X, Y) > v\}$ .
- 2. Let X and Y have the bivariate Normal distribution with  $\mu_X = 15, \mu_Y = -4, \sigma_X = 3, \sigma_Y = 2$  and  $\rho_{X,Y} = -\frac{7}{9}$ . Compute:
  - (a) Pr(X Y > 20) hint: first find the mean and standard deviation of X Y,
  - (b)  $\Pr(X < 12.3 | Y = -2.1).$
- 3. Consider a RIS of size 12 from a Normal distribution with  $\mu = 20$  and  $\sigma = 3.7$ . Compute
  - (a)  $\Pr\left(|\bar{X} 20| < \frac{s}{3}\right)$ ,
  - (b) Pr(s > 4), where  $\overline{X}$  and s are the corresponding sample mean and sample standard deviation respectively.
- 4. Let  $X_1$ ,  $X_2$ ,  $X_3$ ,  $X_4$  and  $X_5$  be a RIS of size 5 from the exponential distribution with the mean of 8. Compute:
  - (a)  $Pr(\overline{X} > 6)$ , where  $\overline{X}$  is the corresponding sample mean (hint: we know the distribution of the *sum* of the five Xs),
  - (b)  $\Pr\{X_1 + X_2 + X_3 > 2X_4 + 2X_5\}$ . Hint: we know the distribution of  $\frac{X_4 + X_5}{X_1 + X_2 + X_3 + X_4 + X_5}$ .
- 5. Let X and Y be two RVs having the following joint PDF:

$$f(x,y) = \begin{cases} 10x^2 + 4xy & \text{when } x > 0, \ y > 0 \text{ and } x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find

- (a) the conditional PDF of X given that  $Y = \frac{1}{3}$  (do not forget to stipulate its support),
- (b) the mean and the *median* of Y (hint: use the marginal distribution of Y).