

Full credit given for 5 (out of 10) correct and complete answers.

Numerical answers should be given in **decimal**.

Final (brief) answers **must** be entered in your booklet.

Send your Maple to jvrbik@brocku.ca

Open-book exam.

Duration: 1 hour

1. Let X be a RV with the following pdf

$$f(x) = \begin{cases} \frac{1}{2} & -1 < x < 0 \\ 1 - x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find

- (a) the pdf (including its support) of

$$Y = X^2$$

- (b) and the variance of (the same) Y .

2. Let X and Y have the bivariate Normal distribution with $\mu_x = 2.8$, $\mu_y = -1.2$, $\sigma_x = 1.3$, $\sigma_y = 2.4$ and $\rho = -0.74$. Find

- (a) the MGF of $3X - 2Y + 4$,

- (b)

$$\Pr(X < 2 | Y = -1)$$

3. Consider a RIS of size 10 from Normal distribution with $\mu = 2$ and $\sigma = 0.4$. Compute

- (a)

$$\Pr \left[\left(20 - \sum_{i=1}^{10} X_i \right)^2 < \sum_{i=1}^{10} (X_i - \bar{X})^2 \right]$$

- (b)

$$\Pr \left(1 < \sum_{i=1}^{10} (X_i - \bar{X})^2 \right)$$

where X_1, X_2, \dots, X_{10} represent the individual observations. Hint: first express the inequalities in terms of \bar{X} and s^2 (partial credit given for this alone).

4. Consider two RVs X and Y having the following joint pdf

$$f(x, y) = \begin{cases} \frac{1}{3}(x + y^2) \exp(-x - y) & \text{when } x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find

(a)

$$\Pr(2X + 5Y < 7)$$

(b) the conditional pdf of Y (including its support), given that X has been observed to have the value of 1.25,

(c) the *median* (not mean!) of Y ,

(d) the pdf (including its support) of

$$U = \frac{X}{X + Y}$$

Hint: when integrating, let Maple know the potential range of U values by ‘assuming ... $< u < \dots$ ’.