MATH 3P85 **Second midterm** **26 March 2018**
Full credit given for 9 (out of 15) correct and complete answers.
Numerical answers should be given in decimal, to 4 significant digits.
Symbolic answers must be fully simplified.
All final (brief) answers **must** be entered in your booklet.
Send your Maple to **jvrbik@brocku.ca** (and keep a copy).
Open-book exam. **Duration: 1 hour**

1. Let X_1 and X_2 be two independent RVs, each having the following pdf

$$f(x) = \begin{cases} \frac{1}{4} & -2 < x < 0 \\ 1 - x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Compute

   (a) $\Pr(X_1 + X_2 < \frac{1}{4})$, hint: use convolution,

   (b) the median and quartile deviation of $X_1 + X_2$,

   (c) $\Pr(X_1 < X_2)$.

2. Let X and Y have the bivariate Normal distribution with $\mu_x = 2.8, \mu_y = -1.2$, $\sigma_x = 1.3$, $\sigma_y = 2.4$ and $\rho = -0.74$. Find

   (a) $\Pr(2X - 3Y < 10)$,

   (b) $\Pr(2X - 5 > 0)$,

   (c) $\Pr(2X - 5 > 0 \mid Y = 0)$,

3. Consider a RIS of size 10 from Normal distribution with $\mu = 2$ and $\sigma = 0.4$. Compute

   (a) $\Pr\left(\bar{X} - 2 < -\frac{s}{4}\right)$,

   (b) $\Pr\left(s > 0.5\right)$,

   (c) $\Pr\left[\sum_{i=1}^{4}\left(X_i - \frac{\sum_{j=1}^{4} X_j}{4}\right)^2 < \sum_{i=5}^{10}\left(X_i - \frac{\sum_{j=5}^{10} X_j}{6}\right)^2\right]$ (hint: use Fisher-F distribution), where X_1, X_2, ... X_{10} represent the individual observations, and $\bar{X}$ and s are the corresponding sample mean and sample standard deviation, respectively.

4. Assuming that two continuous-type RVs X and Y have the following joint pdf

$$f(x,y) = \begin{cases} (x+y)\exp(-x-y) & \text{when} \quad x > y > 0 \\ 0 & \text{otherwise} \end{cases}$$

find

   (a) the expected value of $X+Y$ and the corresponding standard deviation,

   (b) the pdf of $U = \exp(-X)$ and the corresponding support,

   (c) the joint MGF of X and Y, i.e. $\mathbb{E}\left(\exp(t_1 X + t_2 Y)\right)$, assuming that both t_1 and t_2 are smaller than 1.

5. Consider a RIS of size 11 from a distribution with the following pdf

$$f(x) = \frac{1}{x^2} \qquad \text{when} \quad x > 1$$

(zero otherwise). Compute

   (a) the expected value of the sample *median* and the corresponding standard deviation,

   (b) the probability that the sample median will be bigger than 3,

   (c) the probability that the sample *range* (i.e. $X_{(11)} - X_{(1)}$) will be bigger than 100.