MATH 3P85 FIRST MIDTERM Feb. 12, 2018
Full credit given for 10 (out of 15) correct answers.
When more answers given (good idea), the best 10 count.
Give decimal answers to at least 4 significant digits.
Enter all final answers (usually a single number) in the booklet.
Email your Maple to jvrbik@brocku.ca (and keep a copy).
Open-book exam. Duration: 1 hour

1. There are *three* indistinguishable boxes, *two* of them containing 5 red and 7 blue marbles, the last *one* having 7 red and 5 blue marbles. One of these three boxes is randomly selected and 4 marbles are drawn from it (without replacement).

   (a) Compute the probability that 3 of these marbles are red and 1 is blue.

   (b) Given that the event of Part (a) [i.e. getting 3 red and 1 blue marble] has happened, find the conditional probability that it was the last box which had been selected.

   (c) Given that the event of Part (a) has happened and we proceed to draw yet another marble (4 are already gone) from the same (already selected) box, what is the (conditional) probability that this marble also turns out to be red?

2. Assuming A, B, C and D to be mutually *independent* events with $\Pr(A) = 0.62$, $\Pr(B) = 0.71$, $\Pr(C) = 0.38$ and $\Pr(D) = 0.55$, compute the probability that

   (a) *at least one* of the four events happens (when the corresponding random experiment is carried out),

   (b) *exactly three* of the them happen,

   (c)
$$\Pr[(\overline{A} \cap B \cap \overline{C} \cap D) \cup (A \cap \overline{B} \cap C \cap \overline{D})]$$

3. A random variable X has the following pdf:

$$f(x) = \begin{cases} \dfrac{2x}{5} & 0 \le x < 1 \\ \dfrac{x+1}{5} & 1 \le x < 2 \\ \dfrac{9-3x}{5} & 2 \le x < 3 \\ 0 & \text{otherwise} \end{cases}$$

   Find

   (a) the expected value and standard deviation of $3X - 5$,

(b)
$$\mathbb{E}\left(\frac{X}{1+X^2}\right)$$

(c)
$$\Pr\left(\ln X > \frac{1}{2}\right)$$

4. Consider the following game: a player rolls two dice and receives \$5 when getting more than 10 dots, \$1 when getting between 8 and 10 dots (inclusive), but he has to pay \$2 when the number of dots is 6 or less. He plays 20 rounds of this game. Compute

   (a) the probability that he wins \$5 (individually, not in total) in fewer than 3 of these 20 rounds,

   (b) the expected value and standard deviation of his *total* (20 round) net win,

   (c) the probability that he wins (in total) at least \$10.

5. Assuming that two continuous-type random variables X and Y have the following joint pdf
$$f(x,y) = \begin{cases} 4(x+2y)\exp(-2x-y) & \text{when} \quad x > y > 0 \\ 0 & \text{otherwise} \end{cases}$$
   find

   (a) their covariance,

   (b)
$$\Pr(3X + 2Y > 4)$$

   (c) marginal pdf of Y and its support.