MATH 4F83

Assignment # 3

1. Consider a branching process with two initial members, and the number of offsprings having the binomial distribution with $n = 5$ and $p = 0.23$. Compute the probability of ultimate extinction of this process. Also, find the expected number of members of the 10^{th} generation and the corresponding standard deviation.

2. Find the mean and standard deviation of X, and also $\Pr(X = 0)$ and $\Pr(X = 1)$ assuming that X has a distribution defined by the following PGF:
   a. $F(s) = (0.2 + 0.8\, e^{s-1})^3$

   b. $F(s) = \exp(3s^2 - 3)$

   c. $F(s) = 3\frac{(4-s)^2}{(5-2s)^3}$.

3. Consider a branching process with the following distribution of the number of offsprings:

$X =$	0	1	2	3
Pr:	0.27	.42	0.18	0.13

   The process starts with 7 initial members. Find:
   a. The expected value and standard deviation of the process three generations later.

   b. Probability that the process becomes extinct during the first three generations.

   c. Probability of the process' ultimate extinction.

4. Consider a branching process where the distribution of offsprings is Poisson with the mean of 1.43. The process starts with 4 initial members (the zeroth generation). Compute:
   a. The expected value of Y_5 (the process' progeny, up to and including Generation 5) and the corresponding standard deviation.

   b. Probability of the process' ultimate extinction.

   c. Probability that the process becomes extinct going from the second to the third generation (i.e. due to the second generation having no offsprings).

5. Suppose that each bacteria of a specific strain produces, during its lifetime, a random number of offsprings, whose distribution is Binomial with $n = 5$ and $p = 0.15$. If we start with a 'culture' containing 2000 such bacteria, calculate the mean and the standard deviation of the total number of bacteria ever produced (including the original batch).

6. Consider betting repeatedly \$3 on a flip of a coin. What is the probability of breaking even for (exactly) the third time in the 12^{th} round of this game?

7. Find the expected number of rolls of a die, and the corresponding standard deviation, to generate the pattern
   a. 6EE6E (6 means six, E means anything else),
   b. 5 consecutive sixes.

8. Find the expected number of flips of a coin and the corresponding standard deviation to generate the pattern HTTHHT
   a. from scratch,
   b. given that the first 3 flips have resulted in HTT (and counting how many $more$ it takes) - this time, get the expected numer only.

9. Calculate the probability of getting 3 consecutive sixes before 8 consecutive nonsixes. What is the expected duration and the corresponding standard deviation of such a game.

10. If the pattern HTTH is played against TTT, find its probability of winning. Also find the expected duration of the game (in terms of number of flips) and the corresponding standard deviation.