1. Consider Continuous-Time Markov Process with the following infinitesimal generator:

$$\begin{bmatrix} \times & 3 & 2 \\ 4 & \times & 1 \\ 3 & 0 & \times \end{bmatrix}$$

(the rates are per hour). Find the expression for $\Pr\{X(t) = 2 \mid X(0) = 2\}$ - assume that the states are labelled 1, 2 and 3.

If the process starts in State 2, what is the probability of being in the same state 15 minutes later? What is the probability that, 15 minutes later, the process is still in State 2 (i.e. without ever leaving it – the answer must obviously be smaller than the previous one – why)?

2. Similarly, if the infinitesimal generator is

$$\begin{bmatrix} -4 & 1 & 3 \\ 0 & -2 & 2 \\ 3 & 1 & -4 \end{bmatrix}$$

(per hour), find $\Pr\{X(t) = 2 \mid X(0) = 3\}$. Evaluate this expression at $t = 0.5$ hour. Also, find the stationary distribution of this process.

3. And one more time: Assuming the infinitesimal generator is

$$\begin{bmatrix} \times & 3.1 & 4.2 \\ 2.8 & \times & 1.9 \\ 3.3 & 3.8 & \times \end{bmatrix}$$

find:

a. $\Pr\{X(1) = 3 \mid X(0) = 3\}$,

b. the stationary distribution of this process.

4. Consider one-dimensional Brownian motion with no drift, an absorbing barrier at zero, and $c = 3\ \frac{\text{cm}^2}{\text{sec}}$, starting at $X(0) = 4$ cm. Calculate the probability of:

a. the process getting absorbed within the first 10 seconds,

b. $X(1 \text{ min}) > 10$ cm.

5. Similarly, assuming a Brownian motion with $c = 13.8\frac{\text{cm}^2}{\text{hr.}}$ and $d = 0$ (no absorbing barrier), find:

a. $\Pr\{X(3 \text{ hours}) > -4 \text{ cm} \mid X(0) = 1 \text{ cm}\}$,

b. $\Pr\{X(24 \text{ hours}\} > 15\text{cm} \cap \min_{0< t <1 \text{ day}} X(t) > 0 \mid X(0) = 3 \text{ cm}\}$,

c. $\Pr\{\max_{0< t <1 \text{ day}} X(t) > 15 \text{ cm} \mid X(0) = 0\}$.

6. Consider the following autoregression model:

$$X_n = 0.9\, X_{n-1} - 0.6\, X_{n-2} + 0.3\, X_{n-3} + \varepsilon_n$$

where $\varepsilon_n \in \mathcal{N}(0, 13)$. Find:

a. the first five (up to and including ρ_5) serial correlation coefficients,

b. the corresponding power spectrum,

c. Var(X_n),

d. the value of the following partial correlation coefficient

$$\rho(X_n, X_{n-3} |\, X_{n-1})$$