

1. Consider a Pure-Death Markov process with  $\mu_n = 2.34 \times n$  per hour, and the initial value of  $X(0) = 13$ . Find:
  - a.  $\Pr\{X(36 \text{ min}) \leq 5\}$
  - b.  $\mathbb{E}[X(16 \text{ min } 37 \text{ sec})]$  and the corresponding standard deviation.
  - c. Probability that the process will become extinct *during* its second hour.
  - d. Expected time till extinction.
2. Consider a Linear-Growth-with-Immigration process. The individual average life-span is 3.12 days, the birth rate is 0.24 per day per each living member, immigration takes place at the rate of 1.17 per day, and the process start in State 0. Find:
  - a. The long-run average value of the process, and the corresponding standard deviation.
  - b.  $\Pr\{X(2 \text{ days}) \geq 3\}$ .

3. Consider a Linear-Growth process *without* immigration and the rates given by

$$\lambda_n = 3 \times n \quad \text{per hour}$$

$$\mu_n = 4 \times n \quad \text{per hour}$$

If this process starts with 2 initial members, what is the probability that it becomes extinct during the next 20 min?

4. Consider a M/M/1 queue with 17.3 arrivals per hour (on the average), the mean service time of 4 min 26 sec, and the probability that an arrival joins the system given by  $0.62^k$ , where  $k$  is the number of customers present (counting the one being served). Find:
  - a. The server utilization factor.
  - b. Percentage of lost customers.
  - c. The average size of the line up.
  - d. Percentage of time with more than two customers waiting for service.
5. Consider another M/M/1 queue with customers arriving at a rate of 12 per hour, and the average service time being 10 min. Also, a customer who arrives and finds  $k$  people waiting for service walks away with the probability of  $\frac{2k}{2k+1}$ . Determine the stationary distribution of this process. What percentage of time will the server be idle, in the long run?
6. Consider a Birth-and-Death process with the following rates

$$\lambda_n = 27 - 3n \quad \text{per hour}$$

$$\mu_n = 5n \quad \text{per hour}$$

where  $n = 0, 1, 2, \dots, 9$ .

- a. If the process starts in State 4, what is the probability that 8 hour later the process is in State 5?

b. What is the expected time between two consecutive visits to State 0 (entry to entry)?

7. Consider another Birth-and-Death process, with rates given by

$$\begin{aligned}\lambda_n &= n\lambda \quad \text{for } n \geq 0 \\ \mu_n &= \begin{cases} 0 & \text{when } n = 0 \\ \mu & \text{when } n \geq 1 \end{cases}\end{aligned}$$

Find an expression for the probability of ultimate extinction, assuming that the process starts in State 3.