

1. If the Math offices is getting phone calls at the rate of 4.7 per hour and the Physics office (independently) at the rate of 5.2 per hour, compute the probability of the Math office getting its 6<sup>th</sup> phone call of the day before the Physics office gets its 4<sup>th</sup> (both offices open at 8 am). Also, what is the expected number and the corresponding standard deviation of phone calls the Physics office gets since opening until the time of the 8<sup>th</sup> phone call to the Math office (Hint: This is Poisson process of random duration).
2. For a Pure-Death process with  $\mu_n = 12.3 \times n$  per hour and 7 initial members, find:
  - (a) Probability that 10 minutes later there will still be more than 2 members left.
  - (b) Probability that 10 minutes later the process is already extinct.
  - (c) Expected time till extinction.
3. Consider the Linear-Growth-With-Immigration process with  $\lambda = \mu = 3.6$  per hour,  $a = 10.8$  per hour, and no initial members. Find the probability that 15 hours later the process will have more than 80 members. Exactly 80 members.
4. For a Birth-and-Death process with

$$\begin{aligned}\lambda_n &= 1.9 \times n \text{ per hour} \\ \mu_n &= 2.1 \times n \text{ per hour}\end{aligned}$$

and 5 initial members, find:

- (a) The expected value of the process one hour later and the corresponding standard deviation.
  - (b) The expected time till extinction.
5. Consider a  $M/M/3$  queue with customers arriving at the rate on 13.7 per hour and having a average service time of 10 minutes. Assuming that the facility has been running for a long time, find:
    - (a) Percentage of time with all three servers busy.

- (b) The average number of customers waiting for service.
- (c) Server utilization factor (assuming equal utilization).

6. Assume that a Birth-and-Death process has the following (per hour) rates

State:	0	1	2	3	4
$\lambda_n$	2.1	4.1	3.0	2.7	0
$\mu_n$	0	5.0	2.4	6.4	3.2

- (a) Find the corresponding stationary distribution and its expected value.
- (b) What is the expected length of a 'busy period' (which starts when the process leaves State 0, and ends when the process comes back to it).

7. Find the general solution to

$$\dot{P}(z, t) = (1 - z)P'(z, t) + P(z, t)$$

Secondly, find the solution to the previous PDE which also meets

$$P(z, 0) = z$$