1. Consider a $M/G/\infty$ queue with service times being uniformly distributed over the 5 to 15 minute interval, and customers arriving at the average rate of 14.7 per hour. Assuming that the process starts in State 0, find the probability that

   (a) half an hour later, more than 4 customers have been already served (and have left), while exactly 3 are still being processed,

   (b) the average number of customers served, per hour, in a long run,

   (c) the percentage of time (in a long run) with fewer than 5 servers busy.

2. Consider a Birth and Death process with $\lambda_n = 3.7 \times n$ per minute, $\mu_n = 3.8 \times n$ per minute, and 3 initial members. Compute:

   (a) Probability that 4 minutes later the process will have at least 2 members.

   (b) Probability that 4 minutes later the process is already extinct.

   (c) Expected time till extinction.

3. Consider an $M/M/\infty$ queue with the average service time of 12 minutes, and customers arriving at the rate of 13.9 per hour. When we start observing the system, there are 4 customers being serviced. What is the expected value and standard deviation of the number of customers in the system five minutes later. What is the long-run percentage of time with all servers idling. How frequently does this happen (i.e. what is the average time between two such consecutive occurrences, measured from the beginning of one, to the beginning of the next).

4. Consider a $M/M/4$ queue with customers arriving at the average rate of 7.3 per hour, and service time taking, on the average, half an hour. Find:

   (a) The average number of busy servers.

   (b) The average number of customers waiting for service.

   (c) The (long run) percentage of time with no line up.

5. Assume that a Birth-and-Death process has the following (per hour) rates

State:	0	1	2	3	4	5
λ_n	4.6	4.3	4.0	3.7	3.1	0
μ_n	0	2.9	3.1	3.6	4.2	4.9

   (a) Find the corresponding stationary distribution, its mean and standard deviation.

   (b) How many times, on the average (in a long run), will State 3 be visited each hour.

6. Make State 0 of the previous example absorbing (by changing λ_0 to 0). Find the expected time till absorption, assuming the process starts in State 5. What is the expected number of visits to State 3 (this may require Maple - if not available, describe in detail how would you proceed).

7. Find the general solution to

$$\dot{P}(z,t) = -\frac{P'(z,t)}{2\,z} + \frac{P(z,t)}{z^2}$$

Secondly, find the solution to the previous PDE which also meets

$$P(z,0) = \frac{z^2}{\exp(z^2) - 1}$$