

BROCK UNIVERSITY

Progress Examination: December 1999
 Course: MATH 4F21
 Date of Examination: Dec. 14, 1999
 Time of Examination: 14:00 -17:00

Number of Pages: 3
 Number of students: 3
 Number of Hours: 3
 Instructor: J. Vrbik

This is an open-book exam.
Full credit given for 7 complete answers.

1. Find

$$\begin{bmatrix} 0 & .3 & .7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ .2 & .2 & .1 & .2 & .3 \end{bmatrix}^{1000}$$

2. Consider a branching process with *three* initial members, and the number of offsprings having the distribution with the following PGF

$$F(s) = \exp(s^2 - 1)$$

- (a) Compute the probability of ultimate extinction of this process.
 (b) Find the expected number of members of the 6th generation and the corresponding standard deviation.
 (c) What is the probability that the first generation will consist of exactly two members?
3. Do the complete classification of the following TPM (\times indicates a nonzero entry, \cdot implies zero):

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \times & \cdot & \times \\ \cdot & \cdot & \cdot & \cdot & \times & \cdot & \times \\ \cdot & \cdot & \cdot & \times & \cdot & \cdot & \cdot \\ \times & \times & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \times & \cdot & \cdot & \times & \cdot \\ \cdot & \cdot & \cdot & \times & \cdot & \cdot & \cdot \\ \cdot & \cdot & \times & \cdot & \cdot & \times & \cdot \end{bmatrix}$$

For each periodic class, find the subclasses.

4. Using the following TPM

$$\begin{bmatrix} .2 & .4 & .1 & .3 \\ .3 & 0 & .3 & .4 \\ .6 & 0 & 0 & .4 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

find $\Pr(X_3 = 2 \cap X_1 = 3)$, given that the initial state is drawn from the distribution

X_0	1	2	3	4
Pr	.25	.35	.25	.15

Also, what is the probability of visiting State 2 before State 3?

5. Find the expected number of flips of a coin and the corresponding standard deviation to generate the pattern HTTHTT.
6. Find the solution to the following difference equation

$$4a_{i+1} - 8a_i + 3a_{i-1} = \frac{3}{2^i} + 1$$

such that $a_{-2} = \frac{209}{9}$ and $a_4 = \frac{195}{8}$.

7. Consider a branching process with *three* initial members, and the number of offsprings having the distribution with the following PGF

$$F(s) = (0.6 + 0.4e^{s-1})^2$$

- (a) Compute the expected value and standard deviation of total progeny of this process.
- (b) What is the probability that Generation 5 is the first one with no members.

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8. Compute the expected number of transitions till absorption and the corresponding standard deviation, given that

$$\mathbb{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ .2 & .2 & .6 & 0 & 0 \\ .3 & .3 & .4 & 0 & 0 \\ .2 & .2 & .1 & .3 & .2 \\ .1 & .1 & .3 & .4 & .1 \end{bmatrix}$$

and the process starts in State 5. Also, what is the expected number of visits to State 3?

9. If the pattern HTHH is played against HHT, find its probability of winning. Also, find the expected duration of the game (in terms of number of flips).
10. Find $\overset{\vee}{\mathbb{P}}$ (the time-reversed TPM) of the following Markov chain

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & .3 & .7 & 0 \\ 0 & .6 & .3 & .1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Is the Markov chain reversible?