BROCK UNIVERSITY

Final Examination: April 2017	Number of Pages: 3
Course: MATH 4F84	Number of students: 19
Date of Examination: April 13, 2017	Number of Hours: 3
Time of Examination: 19:00-22:00	Instructor: J. Vrbik

Open-book exam. Full credit given for 18 (out of 27) complete and correct answers; all must be entered in your booklet; print and attach (or E-mail to jvrbik@brocku.ca) your Maple.

1. Consider a Markov chain with the following TPM - to be able to display matrices of this size, type: interface(rtablesize=12)

$$\mathbb{P} = \begin{bmatrix} 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 \\ 0 & 0 & 0.1 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 \end{bmatrix}$$

(a) Do a complete classification of its states (into recurrent/transient classes and subclasses). Then compute

(b) $\Pr(X_{204} = 4 \cap X_{206} = 8 \mid X_{210} = 5 \cap X_{213} = 5)$ assuming the process is in its stationary mode (hint: consider only the relevant class to answer this question)

(c) the expected number of transitions to reach a recurrent class given $X_0 = 12$, and the corresponding standard deviation,

(d) The following four answers must be given in *fractional* form (partial credit given for decimal values)

$$\lim_{n\to\infty} (\mathbb{P}^n)_{2,3}$$

(e)

$$\lim_{n\to\infty} (\mathbb{P}^n)_{12,3}$$

(f)

$$\lim_{n\to\infty} (\mathbb{P}^{3n+2})_{4,8}$$

(g)

$$\lim_{n\to\infty} (\mathbb{P}^{3n+2})_{12,8}$$

2. Consider a branching process with 13 initial members (in Generation 0), whose offspring distribution has the following PGF:

$$P(z) = \exp\left(\frac{z-1}{2-0.97z}\right)$$

Compute the

(a) expected number of generations till extinction and the corresponding standard deviation,

(b) probability that extinction does not take more than 20 generations,

(c) probability that total progeny will have more than 100 members,

(d) probability that Generation 5 will have more than 25 members.

3. Bob an Alice bet a quarter each on the outcome of the following random experiment: a die is rolled, followed by dealing as many cards (from a standard deck of 52) as the number of dots shown. If no spades are dealt, Bob wins Alice's quarter, any spades dealt and Alice collects his. Compute:

(a) the probability of Bob winning a single round (hint: use the formula of total probability),

(b) the probability of him winning the game, if he starts with \$10.50 (against Alice's measly \$2), and they play till one of them goes broke,

(c) the expected number of rounds (and the corresponding standard deviation) to complete this game,

(d) the probability that the game will end in fewer than 100 rounds (to get the correct answer, first type: Digits:=30).

4. Consider the following difference equation

$$a_{n+2} - 8a_n + 16a_{n-2} = \frac{n^5-1}{n-1} \cdot 2^{n-3}$$

Find

(a) particular solution (hint: factor RHS first),

(b) general solution,

(c) solution which meets: $a_0 = -2$, $a_3 = 3$, $a_6 = 0$ and $a_9 = 4$.

5. Consider rolling a die until generating 3 occurrences of the pattern $6E6EE$ (6 stands for a six, E for anything else). Find

   (a) the expected number of rolls and the corresponding standard deviation,

   (b) the probability that this will take between 100 and 200 rolls (inclusive).

   (c) Compute the expected number of occurrences of this pattern in 400 rolls of a die, and the corresponding standard deviation.

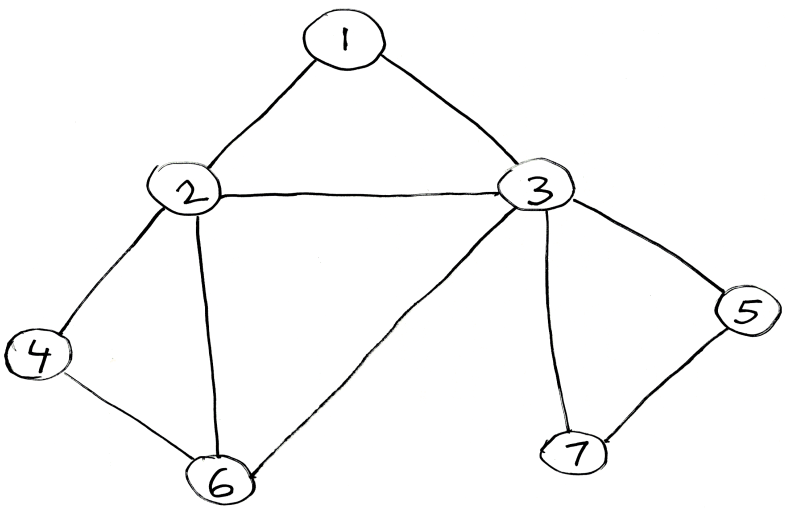
6. Consider rolling a die until either $6E6EE$ or $EE6E6$ appears (for the first time). Find

   (a) the expected number of rolls this game will take, and the corresponding standard deviation,

   (b) the probability that the game will take more than 100 rolls,

   (c) the probability that $6E6EE$ appears before $EE6E6$, thus 'winning' the game.

7. Consider the following random walk



   with the initial state chosen randomly (with the probability of $\frac{1}{7}$ for each state). Find

   (a) the expected number of transitions to reach State 7 (for the first time) and the corresponding standard deviation,

   (b) the conditional probability that 1 was the initial state, given that (exactly) 4 transitions later the process is in State 7,

   (c) the probability of visiting (for the first time) State 4 before visiting State 7.