1. If A, B, C and D are mutually *independent*, and $\Pr(A) = 0.47$, $\Pr(B) = 0.21$, $\Pr(C) = 0.83$ and $\Pr(D) = 0.55$, find

$$\Pr[(A \cap \bar{C}) \cup (\bar{B} \cap D) \cup (A \cap \bar{D})]$$

2. An integer-valued random variable X has the following probability generating function

$$P(z) = \frac{\exp[3.5(z-1)]}{(2-z)^5}$$

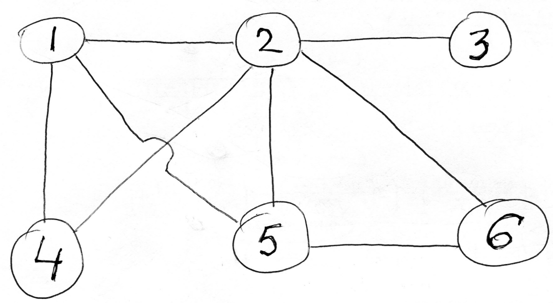
Find

(a) its mean and standard deviation,

(b) $\Pr(10 \le X \le 20)$,

(c) $\Pr(\bar{X} > 10)$, where $\bar{X}$ is the *sample* mean of a random independent sample of size 8 from the corresponding distribution.

3. Consider a random walk over the following network of 6 'nodes'



A 'traveller' starts (at 'time' 0) in Node **1** and then moves, step by step (these are the 'transitions' of the corresponding FMC) into one of the *adjacent* nodes (they all have the same probability of being chosen). Find the probability that

(a) after 3, 5 and 7 transitions the traveller is in Node **2**, **5** and **2** *respectively* (this is *one* question),

(b) the traveller will visit Node **5** (at least once) during the first 6 transitions,

(c) the traveller will visit Node **5** (at least once) during the first 6 transitions without ever (during those 6 transitions) visiting Node **3**.

4. Repeat Question 3 with the traveller being placed (at time 0) into (any) one of the 6 nodes; each node has the same probability of being selected. Note that when we select Node **5** (or **3**) as the initial state, that does count as a visit to that node.

5. For a Markov chain with the following transition probability matrix

$$\mathbb{P} = \begin{bmatrix} 0.12 & 0.21 & 0.13 & 0.23 & 0.31 \\ 0.18 & 0.27 & 0.16 & 0.30 & 0.09 \\ 0.12 & 0.24 & 0.40 & 0.03 & 0.21 \\ 0.28 & 0.13 & 0.12 & 0.06 & 0.41 \\ 0.26 & 0.03 & 0.33 & 0.24 & 0.14 \end{bmatrix}$$

and $X_0 = \mathbf{4}$, find

(a) the corresponding stationary distribution (in exact fractions),

(b) and $\Pr(X_{23} = \mathbf{3} \cap X_{25} = \mathbf{3} \mid X_{20} = \mathbf{1} \cap X_{18} = \mathbf{5})$.