1. Do a complete classification of states of a FMC with the following TPM (a dot implies a non-zero value):

(a)

$$\begin{bmatrix}
\bullet & 0 & 0 & 0 & 0 & 0 & 0 & \bullet \\
0 & 0 & 0 & 0 & 0 & \bullet & 0 & 0 \\
0 & 0 & 0 & 0 & \bullet & 0 & 0 & \bullet \\
0 & 0 & 0 & 0 & 0 & 0 & \bullet & 0 \\
0 & 0 & \bullet & 0 & \bullet & 0 & 0 & 0 \\
0 & \bullet & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \bullet & 0 & 0 & \bullet & 0 \\
\bullet & 0 & 0 & 0 & 0 & \bullet & 0 & 0
\end{bmatrix}$$

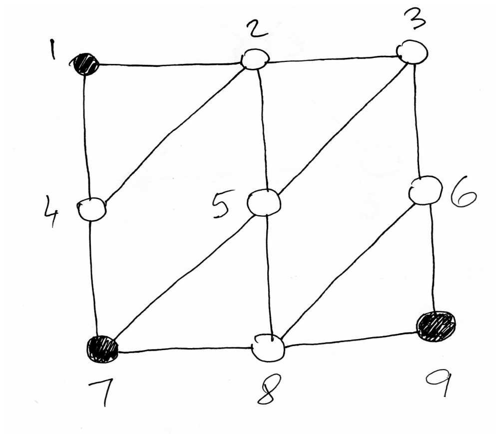
(b)

$$\begin{bmatrix}
\bullet & 0 & 0 & 0 & \bullet & 0 & \bullet & 0 & 0 & 0 \\
0 & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet \\
0 & 0 & 0 & 0 & \bullet & 0 & 0 & \bullet & 0 & 0 \\
0 & 0 & 0 & \bullet & 0 & 0 & 0 & 0 & \bullet & 0 \\
\bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \bullet & 0 & \bullet & 0 & 0 \\
\bullet & 0 & 0 & 0 & 0 & 0 & \bullet & 0 & 0 & 0 \\
0 & 0 & \bullet & 0 & 0 & \bullet & 0 & \bullet & 0 & 0 \\
0 & \bullet & 0 & \bullet & 0 & 0 & 0 & \bullet & 0 & 0 \\
0 & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

(c)

$$\begin{bmatrix}
\bullet & 0 & \bullet & 0 & 0 & \bullet & \bullet & 0 & 0 \\
0 & 0 & 0 & 0 & \bullet & 0 & 0 & 0 & 0 \\
0 & 0 & \bullet & 0 & 0 & 0 & \bullet & 0 & 0 \\
\bullet & \bullet & 0 & \bullet & \bullet & \bullet & 0 & \bullet & \bullet \\
0 & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & \bullet \\
\bullet & 0 & \bullet & 0 & 0 & \bullet & \bullet & 0 & 0 \\
0 & 0 & \bullet & 0 & 0 & 0 & \bullet & 0 & 0 \\
\bullet & \bullet & 0 & \bullet & \bullet & \bullet & 0 & \bullet & \bullet \\
0 & 0 & 0 & 0 & \bullet & 0 & 0 & 0 & 0
\end{bmatrix}$$

2. Consider the following random walk (the full circles are absorbing):



If the walk starts in Node 3, compute

(a) the exact (use fractions) probability of being absorbed, sooner or later, by Node 9,

(b) the expected number of 'moves' till absorption (in any of the absorbing nodes), and the corresponding standard deviation (use decimals from now on),

(c) the probability that absorption will take exactly 5 moves,

(d) the probability that Node 8 will never be visited.

3. For a Markov chain with the following probability transition matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0.11 & 0.13 & 0.32 & 0.21 & 0.23 \\ 0.13 & 0.05 & 0.27 & 0.34 & 0.21 \\ 0.10 & 0.19 & 0.31 & 0.25 & 0.15 \end{bmatrix}$$

and the following initial distribution

State	1	2	3	4	5
Pr:	0.07	0.09	0.26	0.31	0.27

compute:

(a) the exact (a fraction) probability of ending up, eventually, in State 1,

(b) expected value and standard deviation of the number of transitions till absorption (in decimals),

(c) expected number of visits to State 4.

(d) the probability of never visiting State 4.

4. Consider flipping a coin repeatedly until either HTT (consecutive, in that order) or TTH is generated. Compute the

   (a) probability of the first pattern 'winning' (i.e. being generated first) over the second one,

   (b) probability that such a 'game' takes more than 10 flips,

   (c) expected number of flip this game will take, and the corresponding standard deviation.