Exact (i.e. fractional) answers are required in all questions.

1. Without Maple, find the fixed probability vector of the following two TPMs

   (a)

$$\mathbb{P} = \begin{bmatrix} 0 & 0 & 0 & 0.3 & 0.7 & 0 \\ 0 & 0 & 0 & 0.4 & 0.6 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0.3 & 0.5 & 0.2 & 0 & 0 & 0 \end{bmatrix}$$

   (b)

$$\mathbb{P} = \begin{bmatrix} 0 & 0.6 & 0 & 0.4 \\ 0.8 & 0 & 0.2 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0.7 & 0 & 0.3 & 0 \end{bmatrix}$$

2. Using Maple (from now on) find

$$\lim_{n\to\infty} \begin{bmatrix} 0 & 0 & 0.3 & 0.7 & 0 & 0 \\ 0 & 0 & 0.2 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6 & 0.4 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0.3 & 0.7 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^{3n+2}$$

   (hint: this is clearly an example of what we called in class Case 2).

3. Compute

$$\lim_{n\to\infty} \left(\mathbb{P}^{3n}\right)_{4,6}$$

   and

$$\lim_{n\to\infty} \left(\mathbb{P}^{3n}\right)_{3,5}$$

   where

$$P = \begin{bmatrix} 0 & 0.3 & 0 & 0.5 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0.4 & 0 & 0.5 & 0.1 & 0 & 0 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8 \\ 0 & 0 & 0.3 & 0 & 0.3 & 0.4 & 0 & 0 \\ 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 \\ 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0.3 & 0 & 0.6 & 0.1 & 0 & 0 \\ 0 & 0.7 & 0 & 0.2 & 0 & 0 & 0.1 & 0 \end{bmatrix}$$

   (hint: do full clasification of this FMC first, anticipating Case 2 as well).

4. Find

$$\lim_{n\to\infty} (\mathbb{P}^n)_{3,5}$$

and

$$\lim_{n\to\infty} (\mathbb{P}^n)_{8,3}$$

where

$$\mathbb{P} = \begin{bmatrix} 0.2 & 0 & 0.4 & 0 & 0.4 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0 & 0 & 0 & 0.1 & 0.3 & 0.3 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0.7 & 0 & 0 \\ 0.3 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0.7 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(hint: do full clasification of this FMC first - anticipate Case 3).

5. Compute

$$\lim_{k\to\infty} (P^{3k+2})_{4,6}$$

and

$$\lim_{k\to\infty} (P^{3k+2})_{6,4}$$

and

$$\lim_{k\to\infty} (P^{3k+2})_{8,4}$$

where

$$\mathbb{P} = \begin{bmatrix} 0 & 0 & 0.3 & 0.3 & 0.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0.2 & 0.1 & 0 & 0.2 & 0 & 0.3 & 0 \\ 0 & 0.1 & 0.2 & 0 & 0 & 0.2 & 0 & 0.1 & 0 & 0.4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(hint: on closer inspection, the TPM reveals the pattern of Case 4).