1. Find the following limit, using exact fractions

$$\lim_{n\to\infty} \begin{bmatrix} 0 & 0.4 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.7 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 & 0 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.2 \end{bmatrix}^{6n+4}$$

2. Without Maple, solve the following difference equation

$$4a_{n+1} = 9a_{n-1} + (n-2) \cdot \left(\frac{3}{2}\right)^{n+1}$$

subject to the following boundary conditions:

$$\begin{aligned} a_1 &= 2 \\ a_{10} &= 0 \end{aligned}$$

3. Without Maple, find the general solution to

$$a_{n+2} - 2a_{n+1} - 3a_n + 8a_{n-1} - 4a_{n-2} = 2^{n+1} - 3$$

Hint: anticipate a double root of the corresponding characteristic polynomial.

The last two questions should be answered in decimal, with Maple's help.

4. Mr A and Mr B play the following game: 6 cards are dealt (from a shuffeled deck of 52 cards) and if there is more than one spade, Mr A collects \$3 from Mr B, otherwise he loses \$3 to Mr B. Mr A starts the game with \$90, Mr B with \$78, and they agree to play until one of them goes broke. What is the

   (a) probability of Mr A winning the game,
   (b) expected number of rounds they will need to play, and the corresponding standard deviation,
   (c) probability the game will take fewer than 200 rounds.

5. The same two people agree to roll a die and, when it shows more than 4 dots, Mr A pays Mr B \$2; otherwise he collects from him \$1; is this a fair game? If Mr A starts with \$50 in his pocket and Mr B with \$40, what is the probability that Mr. A can play no more (having fewer than \$2 he needs to place his bet) before Mr B loses all his money. Hint: set up and solve the corresponding difference equation.