BROCK UNIVERSITY

Final Examination: April 2004	Number of Pages: 3
Course: MATH 4F84	Number of students: 8
Date of Examination: Apr. 22, 2004	Number of Hours: 3
Time of Examination: 14:00-17:00	Instructor: J. Vrbik

This is an open-book exam. Full credit given for **7** correct and complete answers.

1. Consider a Markov chain with the following transition probability matrix:

$$\mathbb{P} = \begin{bmatrix} 0.14 & 0.31 & 0.29 & 0.26 \\ 0.16 & 0.23 & 0.41 & 0.20 \\ 0.37 & 0.29 & 0.15 & 0.19 \\ 0.24 & 0.09 & 0.52 & 0.15 \end{bmatrix}$$

   Find:

   (a) The stationary probability vector (using fractions).

   (b) $\Pr\{X_2 = 3 \cap X_5 = 1\}$ assuming that X_0 is generated from the following distribution

$X_0 =$	1	2	3	4
Pr	0.22	0.33	0.08	0.37

   (c) Probability of visiting State 4 before State 2 (using the same initial distribution).

2. Consider a branching process having the following distribution for the number of offsprings

$X =$	0	1	2	3	4
Pr	0.31	0.34	0.20	0.10	0.05

   and 5 initial members (in Generation 0). Compute:

   (a) probability of extinction within the first seven generations,

   (b) probability of ultimate extinction,

   (c) expected value and standard deviation of the number of members of Generation 7,

   (d) probability that Generation 7 has between 10 and 20 members (inclusive).

3. Find the expected value and the corresponding standard deviation of the number of rolls of a die to generate the pattern $E6E6$ (6 stands for a six, E for anything else) for the *fifth* time, *allowing* consecutive occurrences to overlap.

4. Find (in terms of exact fractions) the fixed vector of the following transition probability matrix:

$$\mathbb{P} = \begin{bmatrix} 0 & 0.3 & 0.4 & 0 & 0.3 \\ 0.4 & 0 & 0 & 0.6 & 0 \\ 0.7 & 0 & 0 & 0.3 & 0 \\ 0 & 0.5 & 0.1 & 0 & 0.4 \\ 0.5 & 0 & 0 & 0.5 & 0 \end{bmatrix}$$

   Also, construct the probability transition matrix of the time-reversed Markov chain. Based on this, compute $\Pr(X_{503} = 2 \mid X_{507} = 5)$.

5. Consider a branching process with 4 initial members (Generation 0), and the following probability generating function for the distribution of the number of offsprings

$$F(s) = \left(\frac{0.71 - 0.11s}{1 - 0.4s}\right)^2$$

Compute:

   (a) Expected number of members this process will have had up to and including Generation 7, and the corresponding standard deviation.

   (b) Expected number of generations till extinction, and the corresponding standard deviation.

   (c) Expected value of total progeny, and the corresponding standard deviation.

6. Tom and Bob bet a quarter each on the outcome of rolling two dice. Tom wins if neither of the two numbers is bigger than 4, Bob wins otherwise (at least one of them is bigger than 4). Tom starts with \$11.50, Bob with \$2.75 and they agree to play till one of them goes broke. What is the probability of Tom winning the game, the expected number of rounds they will have to play, and the corresponding standard deviation?

7. A die is rolled 70 times. What is the expected number of occurrences of the $E6E$ pattern (now, consecutive occurrences are *not* allowed to overlap) and the corresponding standard deviation. Also, what is the probability of generating between 5 and 10 (inclusive) such patterns?

8. Find (exactly) all elements of

$$\lim_{n\to\infty} \begin{bmatrix} 0 & 0 & 0.12 & 0.88 & 0 & 0 \\ 0 & 0 & 0.31 & 0.69 & 0 & 0 \\ 0.44 & 0.56 & 0 & 0 & 0 & 0 \\ 0.91 & 0.09 & 0 & 0 & 0 & 0 \\ 0.24 & 0.05 & 0.06 & 0.11 & 0.24 & 0.30 \\ 0.18 & 0.06 & 0.02 & 0.10 & 0.41 & 0.23 \end{bmatrix}^{2n}$$

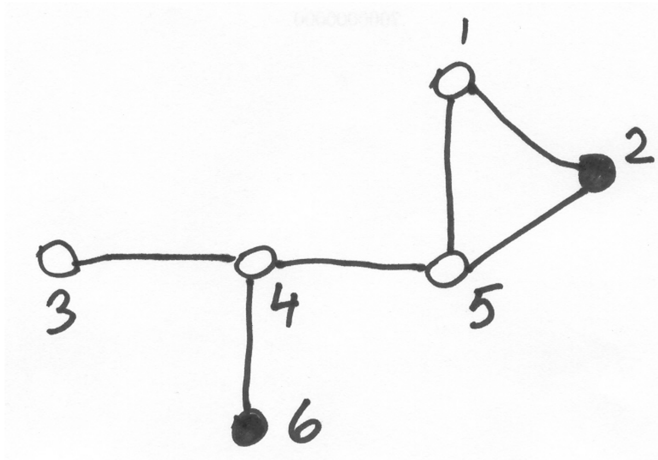
9. Solve the following difference equation

$$9a_{i+2} - 15a_{i+1} + 7a_i - a_{i-1} = i$$

knowing that $a_0 = 3$ and $a_1 = -1$ and $a_2 = 0$.

10. Assume that, in a independent sequence of trials, a success will happen with the probability of 0.62. What is the probability of five consecutive successes winning over three consecutive failures. How many trials does such a game take, on the average, and what is the corresponding standard deviation? What is the probability that this game will be over in fewer than 10 trials?

11. For the following random walk (solid circles are absorbing)



with the initial state chosen randomly (with the probability of $\frac{1}{6}$ for each state) what is the

(a) probability of ending up in State 2,

(b) expected number of moves till absorption, and the corresponding standard deviation,

(c) expected number of visits to State 5.