

MATH 4P84 FIRST MIDTERM OCTOBER 7, 2013

Open book exam. Full credit given for three correct and complete solutions.
Give all answers in fractional form.

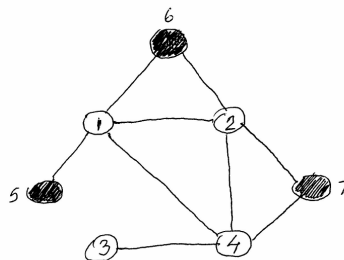
Duration: 50 minutes

1. Consider a FMC with the following TPM

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ \frac{2}{5} & 0 & \frac{3}{5} & 0 & 0 \end{bmatrix}$$

Assuming that the process starts in State 1 (the states are labelled 1 to 5),

- (a) compute the expected number of transitions to reach State 3 (for the first time),
 - (b) find the long-run relative frequency of visits to State 3.
2. Consider the following random walk (dark circles indicate absorbing states):



Assuming that we start in State 3, what is the probability that

- (a) it will take more than 5 ‘moves’ to get absorbed (by any absorbing state),
- (b) the ‘walk’ ends up in State 5.

3. Consider a FMC with the following TPM

$$\mathbb{P} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{3}{7} & 0 & 0 & \frac{4}{7} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix}$$

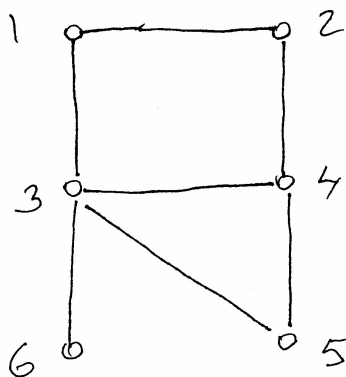
(a) Find

$$\lim_{k \rightarrow \infty} (\mathbb{P}^k)_{6,4}$$

(b) Given we start in State 5, what is the probability of reaching State 3 before reaching State 4?

4. What is the expected number of flips of a coin needed to generate 2 consecutive heads (for the first time)?

5. Consider the following random walk



Assuming that we start in State 1, what is the probability that we will be in State 5

(a) 3 transitions later,

(b) 30,000 transitions later.