MATH 4P84   FIRST MIDTERM   OCTOBER 6, 2014

Open book exam. Full credit given for 6 (out of 10) correct and complete solutions. All answers must be given in fractional form (no decimals).

Duration: 50 minutes
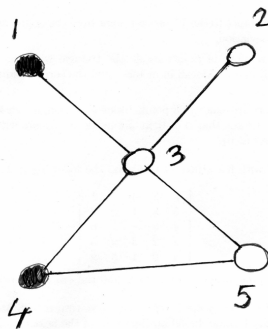
1. Consider a FMC with the following TPM

$$\begin{bmatrix} .30 & .11 & .47 & .12 \\ .21 & .09 & .38 & .32 \\ .27 & .06 & .37 & .30 \\ .48 & .07 & .14 & .31 \end{bmatrix}$$

   (a) Compute
   $$\Pr\left(X_7 = 2 \cap X_5 = 1 \mid X_3 = 2\right)$$

   (b) Assuming that now, the process is in State 4, compute the expected number of transitions to reach State 3 (for the first time from now).

2. Consider the following random walk (dark circles indicating absorbing states):

Assuming that we start in State 2, what is the

   (a) expected number of moves till absorption,

   (b) probability that the walk will end in State 4.

3. Consider a FMC with the following TPM

$$\mathbb{P} = \begin{bmatrix} 0 & 0 & 0 & .55 & .45 \\ 0 & 0 & 0 & .57 & .43 \\ 0 & 0 & 0 & .75 & .25 \\ .35 & .46 & .19 & 0 & 0 \\ .16 & .41 & .43 & 0 & 0 \end{bmatrix}$$

(a) Find
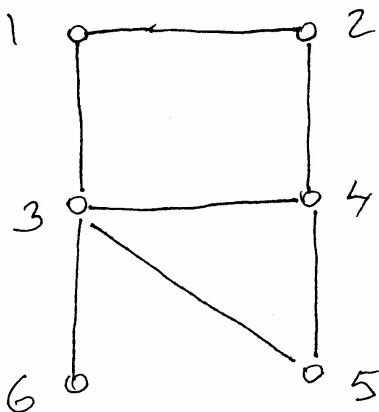$$\lim_{k \to \infty} (\mathbb{P}^{2k+1})_{5,2}$$

(b) Given that we start in State 5, what is the probability of reaching State 3 before reaching State 4?

4. Consider rolling a die till getting two consecutive sixes (for the first time). Compute the

(a) expected number of rolls needed,

(b) and the corresponding variance.

5. Consider the following random walk

Assuming that we start in State 6, what is the

(a) probability of being in State 2 (exactly) four transitions later,

(b) long-run frequency of visits to State 3?