MATH 4P84 FIRST MIDTERM OCTOBER 8, 2015
Full credit given for **6 (out of 10)** correct and complete answers.
Please, give all answers in fractional form.
Open-book exam. Duration: 50 minutes

1. For the following TPM (■ indicates a nonzero value)

$$\begin{bmatrix}
0 & 0 & ■ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & ■ & ■ & 0 & 0 & 0 & ■ & 0 & 0 & ■ \\
0 & 0 & 0 & ■ & 0 & ■ & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & ■ & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & ■ & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & ■ & 0 & 0 & 0 \\
0 & ■ & 0 & 0 & 0 & 0 & 0 & 0 & ■ & 0 & 0 \\
0 & 0 & ■ & 0 & 0 & 0 & 0 & 0 & 0 & ■ & 0 \\
0 & ■ & 0 & 0 & ■ & 0 & ■ & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & ■ \\
■ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

   (a) find all classes and indicate which are recurrent and which are transient,

   (b) for each periodic class, find the corresponding subclasses.

2. Consider a FMC with the following TPM:

$$\mathbb{P} = \begin{bmatrix}
0 & 0 & 0.2 & 0 & 0.8 & 0 \\
0.4 & 0 & 0.6 & 0 & 0 & 0 \\
0 & 0 & 0.1 & 0.5 & 0 & 0.4 \\
0 & 0.7 & 0 & 0 & 0.3 & 0 \\
0.2 & 0 & 0.4 & 0.2 & 0 & 0.2 \\
0 & 0 & 0 & 0 & 1.0 & 0
\end{bmatrix}$$

   and the initial value generated from the following distribution:

$X_0 =$	1	2	3	4	5	6
Pr	0.18	0.13	0.09	0.25	0.20	0.15

   Find:

   (a) $\Pr(X_2 = 3 \cap X_4 = 2)$,

(b) $\lim_{n\to\infty} \Pr(X_n = 5)$.

3. Consider the TPM of the previous question. Is the corresponding FMC lumpable in the following sense? Whenever it is, write down the new TPM.

   (a) 1,3 | 2,5 | 4,6

   (b) 1,6 | 2,3,5 | 4

4. For

$$\mathbb{P} = \begin{bmatrix} 0 & 0 & 0.6 & 0 & 0.4 & 0 \\ 0 & 0 & 0.2 & 0.5 & 0 & 0.3 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0.6 & 0.4 & 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   find:

   (a) $\lim_{n\to\infty}(\mathbb{P}^{2n+1})_{3,2}$

   (b) $\Pr(X_{17} = 3 \cap X_{15} = 6 \mid X_{12} = 1 \cap X_{14} = 2)$.

5. If the following random walk starts in Node 3, find:

   (a) the probability that, four moves later, it is back to Node 3,

   (b) the long run percentage of visits to Node 3.