MATH 4P84	FIRST MIDTERM	FEBRUARY 8, 20017						
Full credit given for 6 (out of 10) correct and complete answers.								
Give all numerical answers in fractional form.								
Open-book exam.		Duration: 1 hour						

1. Consider a FMC with the following TPM (the states are labelled 1 to 10):

	0.2	0.8	0	0	0	0	0	0	0	0]
$\mathbb{P}=$	0	0	1	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0.9	0.1	0	0	0
	0	0	0	0	0	0	1	0	0	0
	0	0	0	0.6	0.4	0	0	0	0	0
	0	0	0	0.5	0.5	0	0	0	0	0
	0	0	0	0	0	0	0	0	0.4	0.6
	0	0	0.2	0	0	0	0	0.8	0	0
	0	0	0	0	0	0	0.3	0.7	0	0

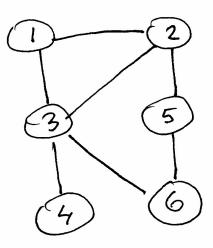
- (a) Find all its classes (spell out the states of each class) and specify which are recurrent and which are transient.
- (b) Identify periodic classes, find their period, and indicate the subclasses.
- (c) Starting in State 5, what is the long-run proportion of visits to State 7.
- (d) Compute $\Pr(X_{105} = 3 \cap X_{103} = 8 \mid X_{100} = 9 \cap X_{98} = 10)$
- (e) and $\lim_{n\to\infty} (\mathbb{P}^n)_{3,1}$.
- 2. Is any one of the following two TPMs lumpable in the following sense: $1, 5 \mid 2, 4 \mid 3$. When the answer is YES, spell out the new TPM; if the answer is NO, give a reason (one is sufficient) why it is not.
 - (a)

$$\begin{bmatrix} \frac{1}{15} & \frac{1}{5} & \frac{1}{3} & \frac{4}{15} & \frac{2}{15} \\ \frac{1}{3} & \frac{1}{5} & \frac{1}{15} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{5} & \frac{1}{15} & \frac{1}{5} & \frac{1}{5} \\ 0 & \frac{1}{3} & \frac{7}{15} & 0 & \frac{1}{5} \\ \frac{4}{15} & \frac{4}{15} & \frac{1}{15} & \frac{2}{15} & \frac{4}{15} \\ 0 & \frac{7}{15} & \frac{1}{3} & 0 & \frac{1}{5} \end{bmatrix}$$

3. Assume that the following random walk starts in either State 1 or State 6 (this is decided by a flip of a fair coin).

Compute the probability that (exactly) 4 transitions later, the process is

- (a) in State 2,
- (b) back to the initial state (which, as we know, is either 1 or 6, depending on the flip's outcome).
- (c) Find the long-run proportion of visits to State 2.



(b)