MATH 4P84 FIRST MIDTERM FEBRUARY 8, 20017

Full credit given for 6 (out of 10) correct and complete answers.

Give all numerical answers in **fractional** form.

Open-book exam. Duration: 1 hour

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1. Consider a FMC with the following TPM (the states are labelled 1 to 10):

$$\mathbb{P} = \begin{bmatrix} 0.2 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.9 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 \\ 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0.7 & 0 & 0 \end{bmatrix}$$

   (a) Find all its classes (spell out the states of each class) and specify which are recurrent and which are transient.

   (b) Identify periodic classes, find their period, and indicate the subclasses.

   (c) Starting in State 5, what is the long-run proportion of visits to State 7.

   (d) Compute $\Pr(X_{105} = 3 \cap X_{103} = 8 \mid X_{100} = 9 \cap X_{98} = 10)$

   (e) and $\lim_{n\to\infty} (\mathbb{P}^n)_{3,1}$ .

2. Is any one of the following two TPMs lumpable in the following sense: $1, 5 \mid 2, 4 \mid 3$ . When the answer is YES, spell out the new TPM; if the answer is NO, give a reason (one is sufficient) why it is not.

   (a)

$$\begin{bmatrix} 0.30 & 0.06 & 0.18 & 0.28 & 0.18 \\ 0.16 & 0.20 & 0.24 & 0.24 & 0.16 \\ 0.14 & 0.09 & 0.08 & 0.26 & 0.43 \\ 0.21 & 0.14 & 0.24 & 0.30 & 0.11 \\ 0.28 & 0.27 & 0.18 & 0.07 & 0.20 \end{bmatrix}$$

(b)

$$\begin{bmatrix} \frac{1}{15} & \frac{1}{5} & \frac{1}{3} & \frac{4}{15} & \frac{2}{15} \\ \frac{1}{3} & \frac{1}{5} & \frac{1}{15} & \frac{1}{5} & \frac{1}{5} \\ 0 & \frac{1}{3} & \frac{7}{15} & 0 & \frac{1}{5} \\ \frac{4}{15} & \frac{4}{15} & \frac{1}{15} & \frac{2}{15} & \frac{4}{15} \\ 0 & \frac{7}{15} & \frac{1}{3} & 0 & \frac{1}{5} \end{bmatrix}$$

3. Assume that the following random walk starts in either State 1 or State 6 (this is decided by a flip of a fair coin).

   Compute the probability that (exactly) 4 transitions later, the process is

   (a) in State 2,

   (b) back to the initial state (which, as we know, is either 1 or 6, depending on the flip's outcome).

   (c) Find the long-run proportion of visits to State 2.