MATH 4P84 FIRST MIDTERM FEBRUARY 7, 2018

Open book exam. Full credit given for 3 (out of 5) correct and complete solutions. All answers must be entered in your booklet and given in fractional form (no decimals). Send your Maple to jvrbik@brocku.ca (and keep a copy).

Duration: 1 hour

1. Do a full classification of a FMC having the following TPM ($\blacksquare$ indicating non-zero elements)

$$\begin{bmatrix} 0 & 0 & \blacksquare & 0 & 0 & 0 & 0 & \blacksquare & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \blacksquare & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \blacksquare & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & \blacksquare & 0 \\ 0 & 0 & 0 & \blacksquare & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & 0 & 0 & \blacksquare \\ 0 & 0 & 0 & \blacksquare & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \blacksquare & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare \\ \blacksquare & 0 & 0 & 0 & 0 & \blacksquare & 0 & 0 & 0 & \blacksquare & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & \blacksquare & 0 & \blacksquare & 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & \blacksquare & 0 & \blacksquare & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \blacksquare & 0 & 0 & 0 & 0 & \blacksquare & 0 & 0 & 0 & 0 \end{bmatrix}$$

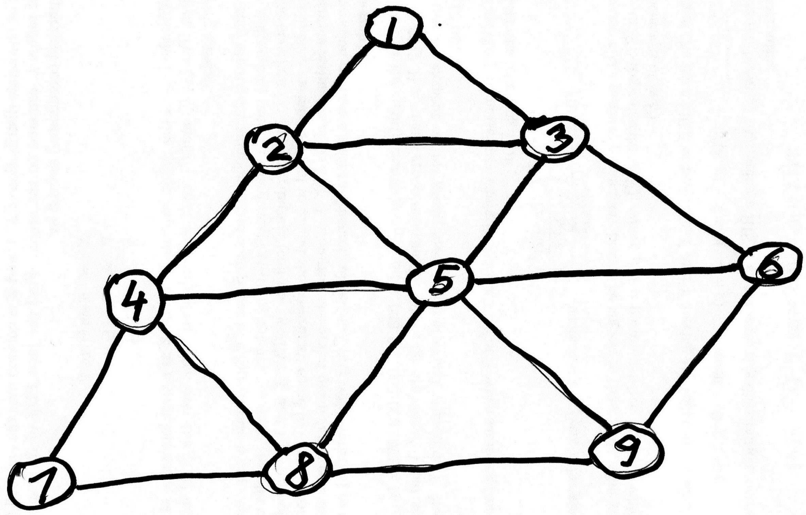
2. Find the expected number of rolls to generate 2 consecutive sixes, and the corresponding variance. What is the probability that this will take longer than 100 rolls (give this answer in decimal, to 4 significant digits).

3. Find the fixed vector of the following PTM

$$\mathbb{P} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ \frac{2}{7} & 0 & \frac{3}{7} & 0 & 0 & \frac{2}{7} & 0 \\ 0 & 0 & 0 & \frac{5}{8} & 0 & 0 & \frac{3}{8} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{5}{14} & 0 & \frac{2}{7} & 0 & 0 & \frac{5}{14} & 0 \\ 0 & 0 & 0 & \frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{bmatrix}$$

and the $\lim_{n\to\infty}(\mathbb{P}^{3n+2})_{2,4}$.

4. Consider a random walk on the following network of nodes



with the initial value chosen randomly (with the same probability for each node). Find the long-run proportion of visits (out of all transitions) to State 7. Also the probability that the process will be back to its initial state (whichever it was) after exactly 3 transitions.

5. Consider the random walk of the previous question, making States 1 and 7 absorbing and starting in State 6. Find the expected number of transitions till absoption (in either of the two absorbing states) and the corresponding standard deviation. What is the probability that

   (a) such an absorption will take more than 10 transitions (express in decimal),

   (b) the 'traveller' gets absorbed in State 7 (back to fractions).