MATH 4P84 FIRST MIDTERM FEBRUARY 5, 20019

Full credit given for 3 (out of 5) correct and complete answers.

All numeric answers must be computed and quoted using *exact fractions.*

Open-book exam. Duration: 1 hour

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1. Do a complete classification of states of an FMC with the following TPM

$$\mathbb{P} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0.4 & 0 & 0.4 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 0 & 0.3 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0.3 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0.3 & 0.2 & 0 \\ 0 & 0 & 0.4 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0.8 \\ 0.1 & 0 & 0 & 0.4 & 0 & 0 & 0 & 0.1 & 0.4 & 0 \end{bmatrix}$$

2. Continuation: compute

   (a)
   $$\Pr(X_{208} = 7 \cap X_{205} = 5 \mid X_{200} = 2 \cap X_{203} = 3)$$

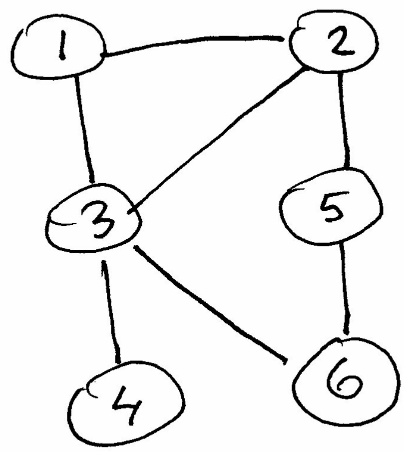
   (b) the probability of visiting State 2 (at least once) *while* avoiding State 7 during the next *five* transitions, given we are currently in State 3 (hint: make State 2 *and* State 7 absorbing).

3. Continuation: compute the following two limits

   (a)
   $$\lim_{n\to\infty} (P^{2n})_{1,9}$$

   (b)
   $$\lim_{n\to\infty} (P^{2n+1})_{6,1}$$

4. Consider a random walk over the following network of nodes, starting in Node 1



Find

(a) the probability of visiting Node 5 before visiting Node 4,

(b) the proportion of visits to Node 5 in a long run.

5. Continuation: if the initial node is chosen randomly (with the same probability for each node), compute

(a) the expected number of transitions till the first visit to Node 5 (choosing Node 5 as the initial node counts as a visit), and the corresponding *variance*,

(b) the probability that the first visit to Node 5 does not take more than 7 transitions.