

MATH 4P84 SECOND MIDTERM NOVEMBER 11, 2013

Open book exam. Full credit given for three correct and complete solutions.
Answers to the first three questions must be exact (i.e. in fractions).

Duration: 50 minutes

1. Find $\lim_{n \rightarrow \infty} (\mathbb{P}^{2n+1})_{5,4}$ where

$$\mathbb{P} = \begin{bmatrix} 0 & 0 & 0.4 & 0.6 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 & 0 \\ 0.6 & 0.4 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0.1 & 0.3 & 0.2 & 0.1 & 0.1 & 0.2 \\ 0.2 & 0.1 & 0.3 & 0.1 & 0.1 & 0.2 \end{bmatrix}$$

2. Consider FMC with

$$\mathbb{P} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.34 & 0.66 \\ 0.29 & 0.71 & 0 & 0 & 0 \\ 0.57 & 0.43 & 0 & 0 & 0 \end{bmatrix}$$

Find a TPM

- (a) of the time-reversed process;
 - (b) having the same fixed vector as \mathbb{P} , but the corresponding Markov chain being time reversible.
3. Find the general solution to

$$a_n - 5a_{n-1} - 8a_{n-2} + 48a_{n-3} = 2^{2n+1} + 2^{3n-1}$$

4. Alice has \$36 and Bob has \$42, and they agree to play the following game, repeatedly, till one of them goes broke: A standard deck of cards is shuffled and 8 cards are dealt; if none of them is an ace, Alice pays \$3 to Bob, otherwise, Bob pays \$3 to Alice (the dealt cards are then returned to the deck, which is reshuffled for the next round.) What is the

- (a) expected number of rounds they will have to play, and the corresponding standard deviation;
 - (b) probability that Alice wins the game (finishing with \$78 in her pocket).
5. Consider a branching process starting with 3 members (Generation 0) and the following offspring distribution:

$X =$	0	1	2	4
Prob:	0.34	0.38	0.23	0.05

What is the

- (a) expected number of members of the 100th generation, and the corresponding standard deviation;
- (b) probability of ultimate extinction of this process.