MATH 4P84SECOND MIDTERMNOVEMBER 17, 2014Full credit given for 6 (out of 10) correct and complete answers.Open-book exam.Duration: one hour

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- Alice and Bob play, repeatedly (until one of them loses all his/her money), the following game: they roll two dice and Alice pays Bob \$3 if the total number of dots shown is more than 7, otherwise she collects \$3 from him. Assuming that she starts with \$15 and Bob with \$135, what is
  - (a) the probability that he ends up with \$150,
  - (b) the expected number (and the corresponding standard deviation) of *dots shown* (from the beginning till either person goes broke).Hint: Recall the *composition* of two distributions.
- 2. For

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$$\mathbb{P} = \begin{bmatrix} 0 & 0 & 0 & 0.3 & 0.7 & 0 \\ 0 & 0 & 0 & 0.2 & 0.8 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0.3 & 0.3 & 0.4 & 0 & 0 & 0 \\ 0.2 & 0.7 & 0.1 & 0 & 0 & 0 \\ 0.2 & 0 & 0.2 & 0 & 0.3 & 0.3 \end{bmatrix}$$

find (in an exact, i.e. fractional form):

(a)

$$\lim_{n\to\infty} \left(\mathbb{P}^{2n+1}\right)_{6,5}$$

(b)

$$\Pr\left(X_{327} = 4 \cap X_{328} = 3 \mid X_{330} = 1\right)$$

Hint: This time, you need to consider the *recurrent* part of  $\mathbb{P}$  only.

3. Consider the following difference equation

$$a_{n+2} - 13a_{n+1} + 39a_n - 27a_{n-1} = (3^{n-1} - 2)(3^{n+1} + 2)$$

Find (again, the exact form of)

- (a) its general solution,
- (b) the value of  $a_{12}$  given that  $a_0 = a_1 = a_2 = 0$ .
- 4. Consider a branching process where the number of offspring of each individual has the following distribution:

# of offspring:	0	1	2	3	4	5
Probability:	0.42	0.34	0.13	0.07	0.03	0.01

Assuming that the process starts with 15 individuals in Generation 0, compute (this time in decimal):

- (a) the expected number of 'truncated' progeny up to and including Generation 7, and the corresponding standard deviation,
- (b) expected 'time' (in terms of generations) till extinction, and the corresponding standard deviation.
- 5. Consider a branching process where the number of offspring of each individual has the following distribution:

# of offspring:	0	1	2	3	4	5
Probability:	0.40	0.35	0.14	0.07	0.03	0.01

Assuming that the process starts with 15 individuals in Generation 0, compute the probability of

- (a) its ultimate extinction,
- (b) the  $7^{th}$  generation having between 12 and 25 members (inclusive).