MATH 4P84   SECOND MIDTERM   NOVEMBER 19, 2015

Full credit given for **5 (out of 10)** correct and complete answers.

Please, give all answers in fractional form (unless specified otherwise).

Open-book exam.   Duration: 1 hour

1. Consider a Markov chain with the following TPM

$$
\begin{bmatrix}
0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.2 & 0.8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.2 & 0 & 0 & 0 & 0 & 0.2 & 0.3 & 0.3 \\
0 & 0 & 0.3 & 0 & 0 & 0 & 0.5 & 0.2
\end{bmatrix}
$$

   Find:

   (a)
$$\lim_{n \to \infty} \Pr(X_{3n+1} = 6 \mid X_0 = 4)$$

   (b)
$$\lim_{n \to \infty} \Pr(X_{3n+1} = 6 \mid X_0 = 8)$$

   (c)
$$\lim_{n \to \infty} \Pr(X_{3n+2} = 2 \mid X_0 = 4 \cap X_{3n} = 3)$$

   (d)
$$\lim_{n \to \infty} \Pr(X_{3n} = 3 \mid X_0 = 4 \cap X_{3n+2} = 2)$$

2. Two people bet \$1 each on a flip of a coin (Mr. A bets on heads, Mr. B on tails). They start with \$73 and \$58 respectively, and agree to play till one of them goes bust. Compute

   (a) the expected number of rounds till the completion of this game, and the corresponding standard deviation,

   (b) the probability that this game will take fewer than 2000 rounds (get the exact answer, but quote it in decimal; it helps to 'simplify' the corresponding PGF first).

3. Without *any* help from 'rsolve'

   (a) find a *general* solution to the following difference equation

   $$2a_{n+2} - 7a_{n+1} + 8a_n - 3a_{n-1} = \left((\tfrac{3}{2})^{n-1} - (\tfrac{2}{3})^{n+1} \right)^2$$

   You must spell out the *form* of each particular solution and show how you established the individual values of 'undetermined' coefficients.

   (b) Find a solution to the previous equation which meets $a_0 = -3$, $a_1 = 2$ and $a_2 = 0$. Compute the value of a_5.

4. Consider the following random walk (solid circles indicate absorbing states).
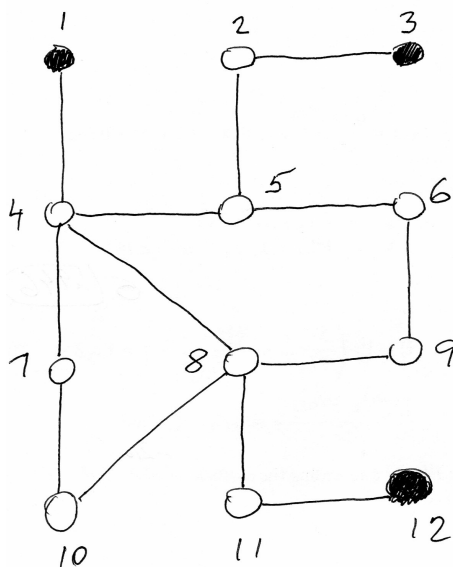
   If the process starts in Node 7, find

   (a) the expected number of moves till absorption and the corresponding standard deviation,

   (b) the probability of getting absorbed without ever returning to Node 7.