MATH 4P84 SECOND MIDTERM MARCH 22, 20017

Full credit given for 3 (out of 6) correct and complete answers.

Answer Q1-4 in **fractional** form, Q5-6 in **decimal**.

Enter **all** (brief) answers in your booklet.

Send your Maple to jvrbik@brocku.ca

Open-book exam. Duration: 1 hour

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1. Using the following stochastic matrix

$$\mathbb{P} = \begin{bmatrix} 0 & 0 & \frac{2}{10} & \frac{4}{10} & \frac{4}{10} & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{10} & \frac{4}{10} & \frac{3}{10} & 0 & 0 & 0 \\ \frac{2}{10} & \frac{8}{10} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{10} & \frac{7}{10} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{4}{10} & \frac{6}{10} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{10} & \frac{2}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{2}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{3}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{2}{10} & \frac{1}{10} & \frac{2}{10} \end{bmatrix}$$

   Find

   (a) $\lim_{n\to\infty}(\mathbb{P}^{2n+1})_{5,1}$

   (b) $\lim_{n\to\infty}(\mathbb{P}^{2n})_{6,1}$

2. Assuming that a FMC with the TPM of

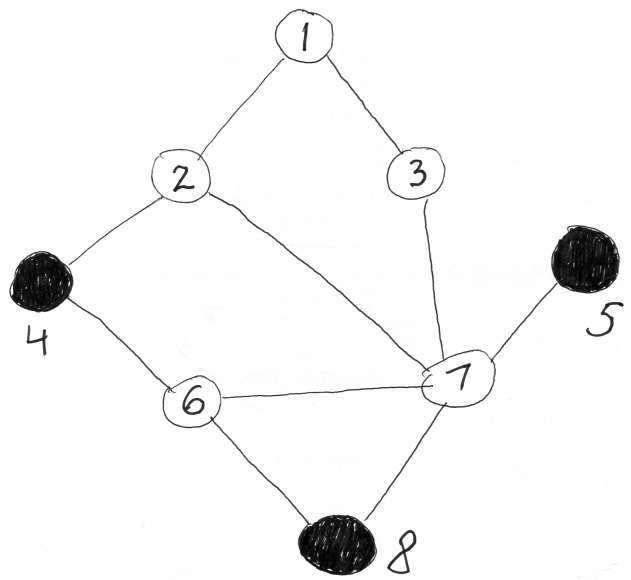
$$\mathbb{P} = \begin{bmatrix} \frac{2}{10} & \frac{3}{10} & \frac{1}{10} & \frac{4}{10} \\ \frac{1}{10} & \frac{5}{10} & \frac{2}{10} & \frac{2}{10} \\ \frac{6}{10} & \frac{1}{10} & \frac{1}{10} & \frac{2}{10} \\ \frac{1}{10} & \frac{7}{10} & \frac{1}{10} & \frac{1}{10} \end{bmatrix}$$

   is running in its *stationary* mode, compute

   (a) $\Pr(X_{234} = 2 \cap X_{237} = 4 \mid X_{239} = 3 \cap X_{241} = 1)$,

   (b) $\Pr(X_{234} = 2 \cap X_{237} = 4)$.

3. Consider a random walk through the following notwork (full circles represent absorbing states), starting in State 1:



Find

(a) the probability of getting absorbed by State 5,

(b) the expected number of transitions till absorption (in any absorbing state) and the corresponding variance.

4. Find a *general* solution to the following difference equation

$$a_{n+2} - 12a_n - 16a_{n-1} = (2^n + n)^2$$

(note the missing a_{n+1} - it is *not* a typo!).

5. Consider a branching process with the following offspring distribution

# of offspring	0	1	2	3
Pr	0.42	0.29	0.18	0.11

which starts (Generation 0) with 12 members. Find

(a) the probability that extinction will take between 12 and 21 generations (inclusive),

(b) the probability that Generation 5 has between 10 and 15 members (inclusive).

6. Two friends agree to bet \$3 on an outcome of a roll of 7 dice; Mr A wins when the dice show at least one 6 *and* at least one 5, Mr B wins otherwise. They agree to play till one of them goes broke (Mr A starts with \$27, Mr B with \$36). Compute

   (a) the probability that Mr A loses all his money,

   (b) the expected duration of this game (in terms of number of rounds) and the corresponding standard deviation.