

Answer as many questions as possible using Maple; highlight all answers.

At **2 p.m.**, email your workspace to **jvr bik@brocku.ca**

Full credit given for **6 (out of 10)** correct and complete answers.

Open-book exam.

Duration: 1 hour

1. For a FMC with the following TPM

$$\begin{bmatrix} 0 & 0 & 0.2 & 0.3 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.4 & 0.2 & 0 & 0 & 0 \\ 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0.3 & 0.3 & 0.3 \\ 0.1 & 0 & 0.2 & 0 & 0 & 0.4 & 0.1 & 0.2 \\ 0 & 0.1 & 0 & 0 & 0.1 & 0.2 & 0.5 & 0.1 \end{bmatrix}$$

find the following two limits (using *exact* fractions)

(a)

$$\lim_{n \rightarrow \infty} (X_{2n} = 4 \mid X_0 = 3)$$

(b)

$$\lim_{n \rightarrow \infty} (X_{2n+1} = 2 \mid X_0 = 6)$$

Note that (only) States 6, 7 and 8 are transient.

2. Consider the following random experiment: Two dice are rolled and the total number of dots recorded. Then a fair coin is flipped till getting the *same* number of *heads* as the number of *dots* obtained in the first part of the experiment. Compute
- the expected number of flips needed, and the corresponding standard deviation,
  - the probability that this will require more than 15 flips.
3. Let two friends bet \$3 each on a flip of a fair coin; the first starts with \$45 in his pocket, the second one with \$54, and they agree to play till one of them runs out of money. Compute

- (a) the probability that the first friend wins this game; also: the expected number of rounds to be played, and the corresponding standard deviation,
  - (b) the probability that this game will take between 100 and 200 rounds (inclusive).
4. Consider a branching process with 8 initial members and the offspring distribution having the following PGF

$$\exp\left(3.5 \cdot \frac{z-1}{5-z}\right)$$

Compute

- (a) the expected number of generations till extinction, and the corresponding standard deviation,
  - (b) the expected progeny up to and including Generation 15, and the corresponding standard deviation.
5. Consider the following difference equation

$$a_{n+2} - 27a_n + 54a_{n-1} = 6^{n-2}$$

- (a) Without using Maple's 'solve' or 'factor', and anticipating a double root of the characteristic polynomial, find the general solution of this equation.
- (b) Using the full power of Maple, and knowing that  $a_0 = 0$ ,  $a_1 = 1$  and  $a_2 = 2$ , find the value of  $a_6$ .