MATH 4P85 SECOND MIDTERM MARCH 24, 2006

Full credit given for three correct and complete answers.

Please, give all answers to four significant digit.

Open-book exam. Duration: 50 minutes

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1. Consider a birth and death process with rates given by

$$\begin{aligned} \lambda_n &= 1.9\,n + 0.6 \qquad \text{per day} \\ \mu_n &= 2.7\,n \qquad\qquad\quad \text{per day} \end{aligned}$$

   and the initial state equal to 5. Find:

   (a) The expected value and standard deviation of the number of members of this process 13 hours later.

   (b) The probability that 13 hours later there are fewer than 5 members left.

   (c) The expected number of visits to State 0 (in a long run) per week.

2. Consider a pure-death process with

$$\mu_n = 0.93\,n \qquad \text{per hour}$$

   starting, at time 0, in State 15.

   Compute:

   (a) The expected value and standard deviation of the number of 'survivals' 55 minutes later.

   (b) The probability that, 55 minute later, there are still more than 7 survivals.

   (c) The expected time till extinction, and the corresponding standard deviation.

   (d) The probability that extinction happens during the third hour (i.e. between $t = 2$ and $t = 3$).

3. Consider an M/M/5 queue, with customers arriving at the rate of 7.3 per hour, and the average length of a service time being 31 minutes. Assuming the process has been running for a long time, compute:

   (a) Percentage of time with more than 6 customers waiting for service.

   (b) The average size of the actual queue (waiting customers).

   (c) The server utilization factor.

   (d) The average time a customer spends in the system (Little's formula tells us that this equals the average number of customers in the system, divided by the arrival rate).

4. Consider an M/M/1 queue with 11 customers arriving on the average every hour, but walking away with the probability of $1 - \frac{1}{\sqrt{n+1}}$, where n is the number of people in the system they find upon their arrival. The average service time is 12 minutes and 17 seconds. Compute:

   (a) The server utilization factor.

   (b) The average length of an idle period (and, of a busy period).

   (c) The mean and standard deviation of the number of customers in the system exactly 2 weeks from now.

5. Solve

$$(z-1)\overset{\bullet}{P} = (z+1)P' - P$$

   (where P is a function of z and t), subject to

$$P(z,0) = e^{z/2}$$