Answers to questions starting with the ■ symbol are to be worked out by hand (no Maple).

1. ■ If $A$, $B$, $C$ and $D$ are mutually *independent*, and $\Pr(A) = 0.47$, $\Pr(B) = 0.21$, $\Pr(C) = 0.82$ and $\Pr(D) = 0.55$, find

$$\Pr[(A \cap \bar{C}) \cup (B \cap D) \cup (A \cap \bar{D})]$$

2. Let $X$ and $Y$ have a bivariate distribution with the following *joint* probability density function (PDF):

$$f(x, y) = \begin{cases} \frac{420}{169}(x + y^2) & \text{when } 0 < x < 1 \text{ and } 0 < y < 1 - x^2 \\ 0 & \text{otherwise} \end{cases}$$

   Find the

   (a) *marginal* PDF of each $X$ and $Y$ (including support),

   (b) correlation coefficient between $X$ and $Y$,

   (c) ■ *conditional* PDF of $X$ given that $Y = \frac{1}{2}$ (including support).

3. An integer-valued random variable $X$ has the following probability generating function

$$P(z) = \exp[(2 - z)^{-5} - 1]$$

   Find

   (a) ■ its mean and standard deviation,

   (b) $\Pr(10 \le X \le 20)$.

   (c) $\Pr(\bar{X} > 4)$, where $\bar{X}$ is a sample mean of 30 independent observations from this distribution.

   (d) Answer the previous question (Part c) using the Normal approximation (a.k.a. the central-limit theorem) *with* continuity correction.

4. Consider paying \$5 to play the following game: 6 cards are dealt randomly from a standard deck of 52 cards, and you get paid \$2 for each spade and \$3 for each ace (\$5 for the ace of spades). Find

   (a) ■ the expected net win (loss) and the corresponding standard deviation,

   (b) and the probability of winning at least \$1 (hint: construct the corresponding PGF first).

5. Find and *identify* the PDF of $X + Y$, where $X$ and $Y$ are independent random variables of the following type:

   (a) $X$ ($Y$) is Normally distributed, with the mean of 3 ($-5$) and the standard deviation of 4 (3) respectively,

(b) $X$ $(Y)$ has a Cauchy distribution with the median of 3 $(-5)$ and quartile deviation of 4 (3) respectively,

(c) $\blacksquare$ both $X$ and $Y$ are drawn from an Exponential distribution with the mean of 2.5.