1. Consider a non-homogeneous Poisson process with the average (per minute) arrival rate at time t (in minutes) given by

$$\lambda(t) = \begin{cases} 1 + \sqrt{t} & 0 \le t < 3 \\ 4 - \dfrac{t}{2} & 3 \le t < 6 \\ 1 & t \ge 6 \end{cases}$$

   Calculate

   (a) the probability of at least 4 arrivals during the 2.3 min $< t <$ 4.1 min interval,

   (b) the expected value and standard deviation of the time (in minutes) of the 7^{th} arrival.

2. Consider a 3D Poisson process with the average density of 5.7 'dots' per meter^3 (meters are our units of length). Find

   (a) the expected distance from the origin to the 10^{th} nearest 'dot', and the corresponding standard deviation,

   (b) the probability of having at least 300 dots in the following region

$$x^2 + y^2 + z^2 - 6x + 8y - 4z < -21$$

3. Consider an $M/G/\infty$ queue where service times are random and independent, and their distribution has the following PDF (t is the service time in hours)

$$g(t) = \frac{5}{(1+t)^6} \qquad t > 0$$

   Customers arrive at the average rate of 8.1 per hour (following the rules of a Poisson process - this goes without saying in all such cases). Find

   (a) the long-run proportion of time with fewer than 5 customers in the system,

   (b) the probability that, 43 minutes after starting the operation (in State 0), fewer that 5 satisfied customers have already finished their service, while at least 2 are still being serviced.

4. Customers arrive at a dollar store at a rate of 7.1 per hour; the distribution of their individual purchases is $Binomial(0.2, 12)$.

   (a) Find the expected *total* purchase of customers who arrive at the store during the next 23 minutes, and the corresponding standard deviation.

   What is the probability

   (b) that this total purchase exceeds \$10,

(c) of getting, during the next 4 hours, at least one 'browser' (someone who buys nothing),

(d) of getting *more* than 10 'buyers' (customers who spend at least \$1) before the *second* browser walks in.

5. Suppose the time that a rural gas station stays open during any given day has the gamma distribution with the mean of 8 hours and the standard deviation of 30 minutes; its customers arrive at an average rate of 7.2 per hour.

   (a) Find the expected number (and the corresponding standard deviation) of customers who will arrive during the gas station's opening hours tomorrow.

   (b) What is the probability that the gas station will serve between 50 and 60 (inclusive) customers tomorrow.