1. Consider a Pure-Birth Markov process with $\lambda_n = 2.43 \times n$ per hour, and the initial value of $X(0) = 3$. Find:

   (a) $\Pr\{X(32\,\text{min}) < 12\}$

   (b) ■ $\mathbb{E}[X(15\,\text{min}\,37\,\text{sec})]$ and the corresponding standard deviation.

2. Consider a Pure-Death Markov process with $\mu_n = 2.43 \times n$ per hour, and the initial value of $X(0) = 31$. Find:

   (a) $\Pr\{X(32\,\text{min}) < 12\}$

   (b) ■ $\mathbb{E}[X(15\,\text{min}\,37\,\text{sec})]$ and the corresponding standard deviation.

   (c) ■ Probability that the process will become extinct *during* its second hour.

   (d) Expected time till extinction and the corresponding standard deviation.

3. Consider a Linear-Growth process with the following rates

$$\begin{aligned} \lambda_n &= 3.1 \times n \quad \text{per hour} \\ \mu_n &= 4.1 \times n \quad \text{per hour} \end{aligned}$$

   and the initial value of 4 members. Find

   (a) the probability that, 40 minutes later, the process will have more than 4 members,

   (b) the expected value and standard deviation of the value of $X(40\text{ min})$,

   (c) ■ the probability that the process becomes extinct during the first 23 min.,

   (d) the expected time till extinction, and the corresponding standard deviation.

4. Consider a Linear-Growth process with the following rates

$$\begin{aligned} \lambda_n &= 4.1 \times n \quad \text{per hour} \\ \mu_n &= 3.1 \times n \quad \text{per hour} \end{aligned}$$

   and the initial value of 4 members. Find

   (a) the probability that, 40 minutes later, the process will have more than 4 members,

   (b) ■ the expected value and standard deviation of $X(33\text{ min})$.

   (c) ■ the probability that the process becomes extinct during the first 23 min.,

   (d) ■ the probability of ultimate extinction.

5. ■ Consider the following PDE

$$z\ \dot{P}(z,t) + e^z \cdot P'(z,t) = 0$$

(a) Find its general solution.

(b) Find the solution which meets

$$P(z,0) = \ln(1+z) - z$$