

Questions with the ■ mark should be answered without Maple.

1. ■ Consider a M/M/5 queue with customers arriving at the average rate of 8.3 per hour, and service time taking, on the average, 28 minutes. Find the long-run
  - (a) server utilization factor,
  - (b) average number of customers waiting for service,
  - (c) average waiting time,
  - (d) proportion of time with no line up.
2. Consider a M/M/1 queue with 17.2 arrivals per hour (on the average), the mean service time of 4 min. 12 sec., and the probability that an arrival joins the system given by  $0.67^k$ , where  $k$  is the number of customers *waiting*. Find the long-run
  - (a) server utilization factor,
  - (b) proportion of lost customers,
  - (c) average number of customers waiting for service,
  - (d) ■ average waiting time.

3. Consider a Birth and Death process with the following (per minute) rates

$$\begin{aligned}\lambda_n &= 3.7 \exp\left(-\frac{n}{5}\right) \\ \mu_n &= \frac{4.1 n}{1 + n^2}\end{aligned}$$

- (a) ■ Verify that this process stationary.
  - (b) Find the long-run average value of the process.
  - (c) Compute the long-run frequency of visits to State 10.
  - (d) Given that the process is now in State 10, what is the expected time to reach (for the first time from now) State 5?
4. Consider a Birth and Death process with the following (per minute) rates

$$\begin{aligned}\lambda_n &= 0.68 \ln(1 + n) \\ \mu_n &= \frac{3.1 n^{1.04}}{1 + n}\end{aligned}$$

Given that the process is now in State 8, find the probability that it will get (sooner or later) trapped in State 0 (note that State 0 is absorbing). If this probability is equal to 1, find the expected time till absorption (starting in State 8).

5. Consider a Birth and Death process with the following (per minute) rates

$$\begin{aligned}\lambda_n &= 0.6\sqrt{n} \\ \mu_n &= \frac{3n}{1+n}\end{aligned}$$

Given that the process is now in State 30, find the probability that

- (a) it will become extinct (reaching State 0),
- (b) after (exactly) four transitions, the process is back to State 30,
- (c) the process will never reach State 10.