MATH 4P85 FIRST MIDTERM FEBRUARY 10, 2006

Full credit given for three correct and complete answers.

Please, give all answers to four significant digit.

Open-book exam. Duration: 50 minutes

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1. Consider a $M/G/\infty$ queue with service times having the following *probability density function*

$$g(x) = \begin{cases} 0 & x < 15 \\ \frac{1}{20} - \frac{(x-30)^2}{4500} & 15 < x < 45 \\ 0 & x > 45 \end{cases}$$

where x is measured in *minutes*, and customers arrive at the average rate of 4.2 per *hour*. Assuming that the process starts in State 0, find

   (a) the probability that 1 hour later, more than 3 customers have been already served (and left),

   (b) the percentage of time (in a long run) with more than 3 customers being served.

2. Consider a two-dimensional Poisson process with $\lambda = 12.3$ 'dots'/meter^2. Find the distribution function of the distance from a 'dot' to its *second* nearest neighbor. Compute the expected value and standard deviation of this distribution.

3. Suppose customers arrive at a rate which starts at 4.7 hour^{-1} when the store opens at 8 am and increases linearly to 8.3 hour^{-1}, reached at noon. The rate than stays fixed at 8.3 hour^{-1} till 2 pm, at which time it starts decreasing linearly to 5.8 hour^{-1}, reached at closing time (5 pm).

   (a) What is the expected number of customers in a single day, and the corresponding standard deviation?

   (b) Compute the probability of getting more than 30 customers between 10 am and 3 pm.

   (c) Find the probability that the third customer of the day arrives before 8:30 am.

4. Suppose that customers arrive at the rate of 6.3 'clusters' per hour, where the size of each cluster has the *following distribution*

Size	1	2	3	4	5
Prob:	.32	.37	.20	.09	.02

   (a) Find the expected number of *clusters* arriving during the next 26 minutes, and the corresponding standard deviation.

   (b) Find the expected number of *customers* who will arrive during the next 26 minutes, and the corresponding standard deviation.

   (c) What is the probability that between 4 and 9 (inclusive) customers arrive during the next 26 minutes?

5. Suppose a gas station with an arrival rate of 8.32 customers per hour is open for a random time T, whose distribution has the following probability density function

$$f(x) = \begin{cases} 0 & x < 8 \\ \frac{12-3(x-10)^2}{32} & 8 < x < 12 \\ 0 & x > 12 \end{cases}$$

   where x is measured in hours.

   (a) What is the expected number of customers served and the corresponding standard deviation?

   (b) What is the probability of getting fewer than 62 customers?