MATH 4P85 FIRST MIDTERM FEBRUARY 27, 2004
Full credit given for three correct and complete answers.
Please, give all answers to four significant digit.
Open-book exam. Duration: 50 minutes

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1. Consider a $M/G/\infty$ queue with service times having the following *probability density function*

$$g(x) = \begin{cases} 0 & x < 15 \\ \frac{x-15}{225} & 15 < x < 30 \\ \frac{45-x}{225} & 30 < x < 45 \\ 0 & x > 45 \end{cases}$$

where x is measured in *minutes*, and customers arrive at the average rate of 3.2 per *hour*. Assuming that the process starts in State 0, find

   (a) the probability that 55 minutes later, more than 2 customers have been already served (and left),

   (b) the percentage of time (in a long run) with all servers idle (no customers).

2. Consider the electronic-counter model with $\alpha = 2.7$ minute^{-1}, $\lambda = 26.3$ per minute. The 'size' distribution is uniform between 32 and 47 units.

   (a) What is the expected value and standard deviation of the value of the process 35 minutes later (assuming we start at 0).

   (b) What is the expected value and standard deviation of the value of the process in its stationary mode (Hint: take $t \to \infty$).

3. Suppose a store's customers arrive at a rate which stays fixed (at 14.2 per hour) till noon, it then jumps to 17.5 during the lunch hour. At 1 pm. it starts decreasing, linearly, from this value (of 17.5) until it reaches the rate of 15.3 at 5 pm.

   (a) Find the probability of fewer than 20 arrivals between noon and 1:20.

   (b) What is the probability that the latest arrival of the day (the store closes at 5) will come between 4:50 and 4:55?

4. Suppose that customers arrive at the rate of 5.3 'clusters' per hour, where the size of each cluster has the *negative binomial* distribution with $k = 3$ and $p = \frac{3}{4}$, *reduced* by 2 (so that the smallest possible value is 1, not 3).

   (a) Find the expected number of *customers* who will arrive during the next 17 minutes, and the corresponding standard deviation.

   (b) What is the probability that this number will be between 3 and 6 (inclusive)?

5. Suppose a gas station with an arrival rate of 7.92 customers per hour is open for a random time T, whose distribution has the following probability density function

$$f(x) = \begin{cases} \frac{3}{10^5}x^4(10-x) & 0 < x < 10 \\ 0 & \text{otherwise} \end{cases}$$

   where x is measured in hours..

   (a) What is the expected number of customers served and the corresponding standard deviation?

   (b) What is the probability of getting fewer than 67 customers?