1. (a) Binomial distribution; its probability function, probability generating function, mean, variance, and random independent sample of size 1000 (compared with theoretical frequencies).
   (b) Geometric; ditto.
   (c) Composition of two distributions (binomial and geometric); explain what experiment does it describe, and how can we generated it (again, 1000 times) and compare with theoretical frequencies.
   (d) Exponential distribution; its pdf, distribution function, mean, variance, moment generating function, mean and variance through MGF.
   (e) Gamma distribution; its pdf, MGF and distribution function. Plot of pdf for $\alpha = 7$ and $\alpha = 70$, the latter compared to Normal distribution with the same mean and variance (the Central Limit Theorem).
   (f) Standardized Normal distribution; its pdf and MGF.
   (g) Convolution of two (or more) distributions, demonstrated with uniform $(0, 1)$ distribution. Sum of three such independent RVs is already assuming the Normal shape (Central Limit Theorem again).

2. Generating (or simulating) the first two hours of a Poisson process (with 6.4 'arrivals' per hour, on the average) by
   (a) adding individual inter-arrival times,
   (b) generating the total number of arrivals first, then their arrival times.
   (c) Changing the rate to 1 arrival per hour, opening the 'store' at 8:00, and exploring the time from 1:00 till the next arrival (many students expect, *incorrectly*, this time interval to equal half an hour)!

3.
   (a) Generating random independent sample from Uniform $(0, 1)$ distribution, using the congruential technique (optional).
   (b) Converting a uniformly-distributed random number to Poisson (or any other discrete) distribution, by means of Distribution-function technique.
   (c) Simulating a non-homogeneous Poisson process (given the rate function) from 8 am to 5 pm.

4.
   (a) Two Poisson processes 'competing'.
   (b) Tree-dimensional Poisson 'process'; the nearest-star distribution.
   (c) $M/G/\infty$ queue; the corresponding two processes (customers being serviced, and those who left already) and related issues.
   (d) Simulating $M/G/\infty$ queue.

(e) 'Cluster' process and the distribution of $X(t)$.

(f) Poisson process of random duration; distribution of the total number of 'customers'.

5.

(a) Yule process; the distribution of $X(t)$.

(b) Simulating a general Birth-and-Death process.

(c) Pure-Death process; the distribution of $X(t)$, and of expected time till 'extinction'; simulation.

6. Linear-Growth process (without immigration).

(a) Verifying the distribution of $X(t)$; the $\lambda = \mu$ limit.

(b) Simulating such process.

(c) Probability of extinction.

(d) Distribution of time till extinction (when certain).

7.

(a) Linear growth *with* immigration; distribution of $X(t)$ and of $X(\infty)$ limit; simulating the first 4 hours of such a process.

(b) $M/M/\infty$ queue; distribution of $X(t)$; simulating the first 4 hours of such a process.

(c) N welders (power-supply process); distribution of $X(t)$; simulating the first 100 minutes of such a process.

8.

(a) Finding stationary probabilities of a *general* Birth and Death process; special case of finitely many states.

(b) Finding probabilities of ultimate extinction of a general B&D process.

(c) When extinction certain, finding the expected time till it happens; special case of finitely many states.

9.

(a) Computing $\exp(\mathbb{A}t)$, where $\mathbb{A}$ is a square matrix, and its $\exp(\mathbb{A}\infty)$ limit.

(b) Constituent matrices of a square matrix; computing an *arbitrary* function of $\mathbb{A}$.

10. Brownian motion (alias Wiener process, or diffusion in one dimension).

(a) Simulating 2000 time steps of this process.

(b) Probability of avoiding absorption, for a specific duration of time.

(c) Probability of avoiding absorption *and* ending up in a specific region.

(d) Two formal proofs (see lecture notes).

11. Autoregressive models.

    (a) Simulating 100 time steps of Markov model; finding its empirical and theoretical correlogram.

    (b) 200 time steps of Yule model; empirical and theoretical correlogram.

    (c) Simulating two (very different), more general (3-α) autoregressive models; their empirical and theoretical correlograms.