MATH 4P85 FIRST MIDTERM FEBRUARY 9, 2004
Full credit given for three correct and complete answers.
Please, give all answers to four significant digit.
Open-book exam. Duration: 50 minutes

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1. Consider a $M/G/\infty$ queue with service times having the Gamma distribution, with the following probability density function

$$g(x) = \frac{x}{16}\exp(-\frac{x}{4})$$

where x is positive and is measured in *minutes*, and customers arrive at the average rate of 15.2 per *hour*. Assuming that the process starts in State 0, find

   (a) the probability that 14 minutes later, more than 3 customers have been already served (and left),

   (b) the percentage of time (in a long run) with all servers idle (no customers).

2. Consider two independent Poisson processes with rates of 6.2 and 8.3 per hour, respectively.

   (a) Compute the probability that, during the next 23 minutes the first process gets more than 4 arrivals, while the second one gets fewer than 3 (please note that this is a *single* question - we want *both* events to happen).

   (b) What is the probability that the first process gets its 4^{th} arrival *before* the second one gets its 3^{rd}.

3. Suppose we have a Poisson process whose arrival rate increases, linearly, from 4.3 to 7.2 between 9 o'clock and noon, it then stays fixed at 7.2 during the lunch hour, drops down to 6.1 at 1 pm. and stays fixed at 6.1 till 5 pm.

   (a) Find the probability of more than 15 arrivals between 11:48 and 1:07.

(b) The expected number of arrivals between 10 am and 11:12 am, and the corresponding standard deviation.

4. Suppose that customers arrive at the rate of 9.3 'clusters' per hour, where the size of each cluster has the following distribution:

Size	1	2	3	4	5
Prob:	.35	.3	.2	.1	.05

   (a) Find the expected number of *customers* who will arrive during the next 27 minutes, and the corresponding standard deviation.

   (b) What is the probability that this number will be greater than 14?

5. Suppose a Poisson process with an arrival rate of 13 per hour is observed for a random time T, whose distribution is Gamma with $k = 9$ and $\beta = 47$ minutes.

   (a) What is the expected number of arrivals and the corresponding standard deviation?

   (b) What is the probability of observing more than 100 arrivals?