MATH 4P85 FIRST MIDTERM FEBRUARY 9, 2012

One sheet of notes, and a Maple workspace (loaded from a memory stick) containing any information, are allowed.

Full credit given for six (out of 10) correct and complete answers.

Please, give all answers to at least four significant digit. **Duration: 50 min.**

1. Customers arrive at an average rate of 42 per hour. Each spends (instantaneously, we assume) a random (from our point of view) number of dollars (this is a dollar store) distributed according to a binomial distribution with $n = 10$ and $p = 0.32$. Find

   (a) the expected amount of money spent by the customers who arrive between 9:15 and 9:52 am, and the corresponding standard deviation.

   (b) the probability that, during the same time (9:15 to 9:52), the store will get at least one 'browser' (a customer who buys nothing).

2. Suppose that an owner of a gas station opens it for service sometime between 8 and 9 am (uniformly distributed), but closes it at 5 pm sharp. Customers arrive at an average rate of 12.3 per hour. Find

   (a) the expected number of *customers* who will be served tomorrow, and the corresponding standard deviation,

   (b) the probability that this number will be bigger than 100.

3. Consider a $M/G/\infty$ queue with service times being independent, each taking between 12 and 35 minutes (uniformly distributed over this interval) and customers arriving at an average rate of 9.6 per hour. Find the probability that, half an hour after opening (with no customers waiting at the door)

   (a) more than 3 customers have left already (having finished their service),

   (b) there are exactly 2 busy servers.

4. (Review question). Consider paying \$10 to play the following game: Nine cards are randomly dealt from a regular deck of 52 cards and you receive \$8 for each ace and \$2 for each face card (J, Q, K). Compute

   (a) the expected value and standard deviation of your *net* win,

   (b) the probability of winning (net) exactly \$16.

5. Consider a 2-dimensional Poisson process with $\lambda = 19$ 'dots' per unit^2. Find the probability that

   (a) the number of 'dots' in the region defined by $x > 1$, $y > 2$ and $x + 2y < 6$ is bigger than 7,

   (b) the second nearest dot to the origin is no farther from it than $\frac{1}{4}$ of a unit length.