MATH 4P85 FIRST MIDTERM FEBRUARY 13, 2007
Full credit given for three correct and complete answers.
Please, give all answers to at least four significant digit.
Open-book exam. Duration: 50 minutes

1. Assuming that customers arrive at a rate which increases, linearly, from 11.3 per hour at 9:00, till noon when it reaches the value of 15.7 per hour. At 12:00, the rate suddenly increases to 17.9 per hour and stays that way for the duration of the lunch hour (till 13:00).

   (a) Find the probability of fewer than 15 arrivals between 11:30 and 12:45.

   (b) If t is the time elapsed since 9:00 (i.e. at 10:00 t will have the value of 1, etc.), find the expression for

   $$\Pr(T_2 < t)$$

   where T_2 is the time of the arrival of the second customer (assume that $0 < t < 3$).

2. Suppose that customers arrive at the rate of 3.8 'clusters' per hour, where the size of each cluster has the following distribution

cluster size:	1	2	3	4
Pr:	0.36	0.42	0.18	0.04

   (a) Find the expected number of *clusters* which arrive during the next 21 minutes, and the corresponding standard deviation.

   (b) Find the expected number of *customers* who arrive during the next 21 minutes, and the corresponding standard deviation.

   (c) What is the probability that the number of *customers* who arrive during the next 21 minutes is between 4 and 9 (inclusive)?

3. Consider a $M/G/\infty$ queue with service times of exactly 7 minutes each, and customers arriving at the rate of 9.3 per hour. Find

   (a) the probability that, half an hour after opening, more than 3 customers have already left (having completed their service),

   (b) the long-run percentage of time with more than three customers being serviced.

4. Customers arrive at a constant rate of 17 per hour, but each of them will make a purchase (instantly, we assume) with the probability of only 72%. Compute the probability that

   (a) during the next half hour, the store gets at least 10 customers who buy something and (at the same time - this is a single question) not more than 5 who won't make any purchase.

(b) by the time the store gets its 7^{th} 'buying' customer, it has had no more than 4 'browsing' ones.

5. Suppose a gas station with an arrival rate of 10.7 customers per hour is open for a random time T, whose distribution is uniform between 8 and 9 hours.

   (a) What is the expected number of customers served during this time, and the corresponding standard deviation?

   (b) What is the probability of serving, during this time, fewer than 70 customers?