MATH 4P85 FIRST MIDTERM FEBRUARY 13, 2008

Full credit given for three correct and complete answers.

Please, give all answers to four significant digits. Duration: 50 minutes

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1. Consider a two-dimensional Poisson process in the x-y plane, with the average density of 'points' of 5.36 per m^2. Find

   (a) the expected distance from the origin to the nearest 'point', and the corresponding standard deviation.

   (b) the probability that fewer than 50 points are inside the region

   $$(x-3)^2 + (y+5)^2 \leq 3$$

   (c) the expected value and the corresponding standard deviation of the number of points inside the triangle with the following vertices: $(3,-2)$, $(3,5)$, $(-2,5)$.

2. Customers enter a store at a rate of 12.8 per hour. Each of them buys (instantly, we assume) a random number of items chosen (independently of the other customers) from the *modified* negative binomial distribution with parameters $k = 3$ and $p = 0.4$ (reminder: number of *failures* to get the 3^{rd} success - note that some customers won't buy any items). Find:

   (a) the expected value and standard deviation of the total number of items bought between 9:18 and 9:41,

   (b) the probability that more than 10 items are bought between 9:18 and 9:41,

   (c) the probability that *all* customers who arrive between 9:18 and 9:41 buy *at least one* item each (hint: remember splitting a Poisson process into two).

3. Consider the $M/G/\infty$ queue with $\lambda = 17.3$ per hour, and service times having a distribution with the following probability density function (where s is measured in minutes):

$$g(s) = \begin{cases} 50s^{-3} & s > 5 \text{ min.} \\ 0 & \text{otherwise} \end{cases}$$

Find

(a) the probability that 38 minutes after opening (with no customers waiting) more than 7 customers have already left, while fewer than 5 are still being serviced.

(b) the long-run proportion of time spent in State 0 (no customers),

(c) the expected duration of each 'visit' to State 0 (hint: effectively, time till the next arrival).

4. The rate of customer arrivals follows, between the hours of 9 and 17 (the store is closed otherwise), the following function

$$\lambda(t) = 20 - \frac{(t-13)^2}{3}$$

Find

(a) the probability of at least 40 arrivals between 11:45 and 13:25,

(b) the probability that the third arrival of the day will happen between 9:17 and 9:32 (hint: subtract the probability of the 3^{rd} customer arriving after 9:32 from the probability of him arriving after 9:17),

(c) the expected time of the third arrival (hint: evaluate $\int_9^{17} t \cdot f(t)dt$ by applying 'evalf' to it, where $f(t)$ is the corresponding PDF).

5. Consider a random variable X whose distribution has the following probability generating function

$$P(z) = \left(\frac{2}{3 - \exp(4z-4)}\right)^{5/2}$$

Find

(a) the expected value and variance of X,

(b) $\Pr(X > 9)$,

(c) $\Pr(\sum_{i=1}^{7} X_i > 9)$, where X_1, X_2, ..., X_7 is a random independent sample of size 7 from the above distribution.