1. $\lambda = 5.36/\text{meter}^2$
   a.
$$\begin{aligned} F(x) &= 1 - \Pr(X > x) = 1 - \exp(-\lambda\pi x^2) \\ f(x) &= F'(x) = 2\lambda\pi x \exp(-\lambda\pi x^2) \\ \mu &= \int_0^\infty x \cdot f(x)dx = 0.2160 \text{ meters} \\ \sigma &= \sqrt{\int_0^\infty (x-\mu)^2 f(x)dx} = 0.1129 \text{ meters} \end{aligned}$$
   b. This is circle of radius $\sqrt{3}$ and area equal to 3π. Thus, $\Lambda = 3\mu\lambda$, and the required probability is
$$\sum_{i=0}^{49} \frac{\Lambda^i}{i!} e^{-\Lambda} = 45.23\%$$
   c. This is a right-angle triangle with sides of 7 and 5 meters, and the area of $\frac{5\times 7}{2} = 17.5$. Thus, $\Lambda = 17.5\lambda = 93.8$, which is also the expected number of points. The corresponding standard deviation is $\sqrt{\Lambda} = 9.685$.

2. Cluster process with $\lambda = 12.8/\text{hour}$, considered for a time interval of $t = \frac{23}{60}$ hours. PGF of cluster size is $P(z) = \left(\frac{0.4}{1-0.6z}\right)^3$, with the mean of $\mu_y = \frac{3}{0.4} - 3$ and variance of $\sigma_y^2 = \frac{3}{0.4}\left(\frac{1}{0.4} - 1\right)$.
   a. The expected number of items bought is
$$\mu_y \lambda t = 22.08$$
   with the corresponding standard deviation of
$$\sqrt{(\sigma_y^2 + \mu_y^2)\lambda t} = 12.43$$
   b. PGF of the number of items bought is $H \equiv \exp\left[\lambda t\left(P(z) - 1\right)\right]$. The probability we need is found by Maple: $1 - eval(mtaylor(H, z, 11), z = 1)$, yielding 82.13%.
   c. The 'browsers' arrive (independently of 'buyers') at the rate of $\lambda_B = \lambda \cdot P(0) = 0.8192/\text{hour}$. The corresponding $\Lambda_B = t \cdot \lambda_B$. Probability of no browsers is thus
$$e^{-\Lambda_B} = 73.05\%$$

3. $\lambda = \frac{17.3}{60}/\text{minute}$. $G(s) = 1 - 25s^{-2}$ when $s > 5$ minutes.
   a. $t = 38$, $p_t = \int_0^t \left[1 - G(s)\right] ds/t = \frac{355}{1444}$, $\Lambda_x = \lambda t p_t$ and $\Lambda_y = \lambda t(1 - p_t)$. Answer:
$$\left(\sum_{i=0}^{4} \frac{\Lambda_x^i}{i!} e^{-\Lambda_x}\right) \cdot \left(1 - \sum_{i=0}^{7} \frac{\Lambda_y^i}{i!} e^{-\Lambda_y}\right) = 50.37\%$$
   b. $\Lambda_\infty = \lambda \int_0^\infty \left[1 - G(s)\right] ds = 2.88\dot{3}$. Answer: $e^{-\Lambda_\infty} = 5.595\%$.
   c. Time till next arrival is $\lambda^{-1} = 3.468$ minutes.

4. $\lambda(t) = 20 - (t-13)^2/3$.

a.

$$\Lambda = \int_{11.75}^{13+25/60} \lambda(t)dt$$
$$1 - \sum_{i=0}^{39} \frac{\Lambda^i}{i!} e^{-\Lambda} = 13.44\%$$

b.

$$\Lambda_1 = \int_{9}^{9+17/60} \lambda(t)dt$$
$$\Lambda_2 = \int_{9}^{9+32/60} \lambda(t)dt$$
$$\sum_{i=0}^{2} \frac{\Lambda_1^i}{i!} e^{-\Lambda_1} - \sum_{i=0}^{2} \frac{\Lambda_2^i}{i!} e^{-\Lambda_2} = 19.05\%$$

c.

$$\Lambda(t) = \int_{9}^{t} \lambda(u)du$$
$$F(t) = 1 - \left(1 + \Lambda(t) + \frac{\Lambda(t)^2}{2}\right) e^{-\Lambda(t)}$$
$$f(t) = F'(t)$$
$$\mu = \int_{9}^{17} t \cdot f(t)dt = 9.200 \text{ minutes}$$

Answer: At 9:09 and 12 seconds.

5.

$$P(z) = \left(\frac{2}{3 - \exp(4z-4)}\right)^{5/2}$$

a.

$$\begin{aligned} \mu &= P'(z)|_{z=1} = 5 \\ \sigma^2 &= P''(z)|_{z=1} + \mu - \mu^2 = 35 \end{aligned}$$

b. $1 - eval(mtaylor(P, z, 10), z = 1)$ which yields 18.75%

c. $1 - eval(mtaylor(P\hat{}7, z, 10), z = 1)$ which yields 97.30%