MATH 4P85 FIRST MIDTERM FEBRUARY 9, 2011

One sheet of notes and a Maple workspace (loaded from a memory stick) containing any information are allowed. Full credit given for six (out of 10) correct and complete solutions. Please, give all answers to four significant digits. **Duration: 50 minutes**

---

1. Consider a two-dimensional Poisson process in the usual x-y space, with the average density of 'points' of 5.36 per meter^2 (let a meter be our unit of distance). Find

   (a) the expected distance from the origin to the second nearest 'point', and the corresponding standard deviation,

   (b) the probability that fewer than 60 'points' are inside the region defined by
   $$x^2 - 6x + y^2 + 10y + 30 \leq 0$$

2. Customers enter a store at a rate of 10.8 per hour; 20% of them will only browse, the rest will spend (individually, and independently of each other) a random amount of dollars, which has, to a good approximation, the gamma$(2, 30)$ distribution. Find:

   (a) the expected value and standard deviation of the total amount of money spend, by all buying customers, between 9:08 and 9:43 (we assume that all purchases are made instantaneously upon arrival),

   (b) the probability that the store has been entered by at least 5 buying customers, before the second browsing customer walks in..

3. Consider the $M/G/\infty$ queue with $\lambda = 12.3$ per hour, and service times having a distribution with the following probability density function (where s is time in hours):
   $$g(s) = \begin{cases} 2s \cdot \exp(-s^2) & s > 0 \\ 0 & \text{otherwise} \end{cases}$$

   Find

(a) the probability that, 11 minutes after opening (with no customers waiting), the number of customers being serviced is the same as the number of customers who have already left,

(b) the mean and standard deviation of the number of customers who arrive during the service time of the fifth customer.

4. Customers enter a store at a (time-dependent) rate of

$$\lambda(t) = \begin{cases} 1.2 + \frac{t}{10} & t < 4 \\ 1.6 - \frac{t}{10} & 4 \le t < 6 \\ 1 & 6 \le t \end{cases}$$

per hour, where t is time (also in hours) since the store's opening. Find

(a) the expected value and standard deviation of the time of the 7^{th} arrival,

(b) the probability that the first customer arrives during the first hour and (meaning $\cap$ - this is a single question) the second one arrives during the first two hours (but not necessarily during the second hour). Hint: Partition the sample space according to the number of arrivals during the first hour.

5. Let X_1, X_2, X_3, ... be independent random variables whose common distribution has the following PGF

$$P_x(z) = \frac{1 + 2z + 3z^2}{6}$$

and let N be a random variable whose PGF is

$$P_N(z) = \frac{z}{(3 - 2z)^4}$$

Compute

(a) the expected value and standard deviation of

$$S_N \equiv X_1 + X_2 + ... + X_N$$

(b) $\Pr(S_N > 20)$.