MATH 4P85 FIRST MIDTERM FEBRUARY 12, 2013

One sheet of notes, and a Maple workspace (loaded from a memory stick) containing any information, are allowed.
Full credit given for six (out of 10) correct and complete answers.
Please, give all answers to at least four significant digit. **Duration: 50 min.**

1. Consider a two-dimensional Poisson process in the usual x-y space, with the average density of 'points' of 3.12 per meter^2. Find
   (a) the expected distance from the origin to the third nearest 'point', and the corresponding standard deviation,
   (b) the probability that more than 40 'points' are found inside the region defined by
   $$3x^2 + 5x + 3y^2 - 11y \le 0$$
2. Customers enter a dollar store (everything costs a dollar) at a rate of 14.6 per hour; one quarter of them will only browse, the rest will spend (individually, and independently of each other) a random amount of dollars, which has, to a good approximation, the negative binomial distribution with parameters $k = 2$ (nobody spends less than \$2) and $p = \frac{1}{5}$. The store opens at 8:00. Find the probability that:
   (a) the store has been entered by at least 7 buying customers, before the third browser walks in,
   (b) the customers who enter the store before 8:30 spend more than \$100 in total.
3. Consider the $M/G/\infty$ queue with $\lambda = 8.3$ per hour, and service times having a distribution with the following probability density function (where s is time in hours):
   $$g(s) = \begin{cases} 6s^2 \cdot \exp(-2s^3) & s > 0 \\ 0 & \text{otherwise} \end{cases}$$
   The facility opens at 8:00am with no customers waiting. Find
   (a) the probability that the 3^{rd} *departure* of the day will happen between 9:00 and 9:12 am.
   (b) the long-run average of busy servers.
4. Customers enter a store at a (time-dependent) rate of
   $$\lambda(t) = \left\{ \frac{4 + 10t + t^2}{1 + 2t^2} \right.$$
   per hour, where t is time (also in hours) since the store opened at 8:00 (i.e. at 8:00 $t = 0$, at 9:00 $t = 1$, etc.). Find

(a) the expected time (please express it the xx:yy:zz form) and standard deviation (give it in minutes and seconds) of the time of the 4^{th} arrival,

(b) the probability of more than 5 arrivals between 9:12 and 10:27.

5. Let X_1, X_2, X_3, ... be independent random variables whose common distribution is

X_i :	1	2	3	4
Pr :	0.36	0.32	0.18	0.14

and let N be a random variable (independent of all the X_i) whose distribution is *modified* negative binomial (counting failures only), with parameters $k = 2$ and $p = \frac{1}{5}$ Compute

(a) the expected value and standard deviation of

$$S_N \equiv X_1 + X_2 + ... + X_N$$

(b) $\Pr(S_N > 8)$.