MATH 4P85 FIRST MIDTERM FEBRUARY 12, 2015

Open book exam. Full credit given for six (out of 10) correct (to four significant digits) and complete answers. Use of Maple permitted.

Duration: 1 hour

1. Consider a two-dimensional Poisson process in the usual x-y space, with the average density of 'points' of 1.36 per meter^2 (let meter be our unit of distance). Find

   (a) the expected distance from the origin to the third nearest 'point', and the corresponding standard deviation,

   (b) the probability that fewer than 10 'points' are inside the triangle with the following vertices: $(-1, 3)$, $(2, -2)$ and $(1, 4)$.

2. Customers enter a looney store (every item costs \$1) at a rate of 10.8 per hour. The amount of dollars each of them spends is (from our point of view) a random variable having the *modified* (i.e. counting 'failures' only) negative binomial distribution with parameters 3 and 0.3. Find:

   (a) the expected value and standard deviation of the total amount of money spent by customers who enter the store between 9:06 and 9:44,

   (b) the probability that each of the customers who enter the store between 9:06 and 9:44 spends at least \$5. Hint: 'split' the process of incoming customers into two, and consider those customers who spend fewer than \$5.

   (c) the probability that the store will be entered (starting 'now') by at least 4 *buying* customers (those who spend at least \$1) before the first *browsing* customer (anyone who spends nothing) walks in. Hint: 'split' the process of incoming customers into two.

3. Consider the $M/G/\infty$ queue with $\lambda = 8.3$ per hour, and service times having a distribution with the following PDF (where s is measured in hours):

$$g(s) = \begin{cases} 1 - \dfrac{s}{2} & 0 < s < 2 \\ 0 & \text{otherwise} \end{cases}$$

   Find

(a) The expected number of customers being in service 28 minutes after opening and the corresponding standard deviation,

(b) the expected number of customers who arrive while the second customer is being serviced, and the corresponding standard deviation. Hint: Recall Poisson process of random duration,

(c) the long-run frequency of visits to State 0 (with no customers) per week (seven 24 hour days).

4. Customers enter a store at a (time-dependent) rate of

$$\lambda(t) = \begin{cases} 11 + 2t & t < 1 \\ \dfrac{13}{t + \ln t} & t \geq 1 \end{cases}$$

per hour, where t is time (in hours) since the store's opening. Find

(a) the expected value and standard deviation of the time of the 4^{th} arrival (give both answers in minutes),

(b) the probability that the first customer arrives during the first five minutes of the process *and* (this is a single question) the second one during the next five minutes (i.e. between 5 and 10 minutes after opening).