MATH 4P85 FIRST MIDTERM FEBRUARY 8, 2016

Open book exam. Full credit given for six correct and complete solutions. Write all your answers (at least the final number, to four significant digits) in your booklet. Duration: 1 hour

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1. Consider a *three*-dimensional Poisson process with the average density of 'points' equal to 5.36 per m^3. Find

   (a) the expected distance from the origin to its *second* nearest point, and the corresponding standard deviation,

   (b) the probability of having at least 10 points inside the region defined by
   $$4 < x < y < z < 6$$

2. Customers arrive at a dollar store at an average rate of 42.8 per hour. The number of items (costing \$1 each) a customer buys (independently of everyone else) has the binomial distribution with $n = 15$ and $p = \frac{1}{5}$. The store opens at 8:00 (nobody is waiting at the door). Find the probability that

   (a) they will get at least 6 'spenders' (customers who buy at least one item) before getting the first 'browser' (who does not buy anything),

   (b) the customers who arrive before 9:30 will spend more than \$200 in total.

3. Consider an $M/G/\infty$ queue with the average arrival rate of 17.3 customers per hour, and service times having a distribution with the following probability density function:
   $$g(s) = \begin{cases} \dfrac{2(s-5)}{125} & 5 < s \le 10 \\ \dfrac{30-s}{250} & 10 < s \le 30 \\ 0 & \text{otherwise} \end{cases}$$
   where s is the service time in *minutes*. The service opens at 9:00 (with no customers waiting at the door). Find

(a) the long-run proportion of time with exactly 6 customers being serviced,

(b) the probability that, by 9:23, more than 2 customers have already finished their service.

4. The rate of customer arrivals varies with time, and is given by the following function

$$\lambda(t) = \frac{20t^2}{10 + t^2}$$

where t is time (since the store's opening) in hours. Find

(a) the expected time of the *fifth* arrival, and the corresponding standard deviation,

(b) the probability of getting at least 3 arrivals during the *second* hour (i.e. between $t = 1$ and $t = 2$).

5. Consider a Poisson process with the constant arrival rate of 12.8 per *hour*, observed for a time interval whose length is random, having the gamma distribution with the first parameter equal to 8 and the second parameter equal to 2 *minutes*. Find

(a) the expected number of arrivals observed during that time, and the corresponding standard deviation,

(b) the probability that, during that time, we get at least 6 arrivals.