MATH 4P85 FIRST MIDTERM OCTOBER 5, 2016

Open-book exam

Full credit given for six (out of 10) correct and complete answers.

Please, give all answers to at least four significant digit. **Duration: 50 min.**

1. Clusters (each consisting of two or more customers) arrive at an average rate of 27.3 per hour. The size of each cluster has a *negative* binomial distribution with $k = 2$ and $p = 0.57$. Find

   (a) the expected number of *customers* who arrive between 9:15 and 9:52 am, and the corresponding standard deviation.

   (b) the probability that, during that time (9:15 to 9:52), the store will get at least one cluster with more than 6 customers.

2. Suppose that an owner of a gas station opens it every day for a random time period which has the gamma(10, 50 minutes) distribution (independent of the past). Customers arrive at an average rate of 12.3 per hour. Find

   (a) the expected number of customers who will be served during the next *two* days, and the corresponding standard deviation,

   (b) the probability that this number will be bigger than 200.

3. Consider a $M/G/\infty$ queue with service times taking exactly 20 minutes each and customers arriving at an average rate of 9.6 per hour. Find

   (a) the probability that, 45 minutes after opening, more than 3 customers will have finished their service already,

   (b) the long-run percentage of time with no customers (all servers idling).

4. Suppose traffic accidents in a certain city can be described as a Poisson process with the average rate of

$$\lambda(t) = 16.3 \cdot \left(2 + \cos \frac{\pi \cdot t}{12}\right) \qquad \text{per } \textit{day}$$

   where t is time in *hours* since noon. Assuming it is currently quarter past two in the afternoon, compute

   (a) the expected time of the 3^{rd} accident from now, and the corresponding standard deviation,

   (b) the probability of at least 4 accidents happening within the next two hours.

5. Consider a 3-dimensional Poisson process with $\lambda = 19$ 'dots' per meter^3. Find the probability that

(a) the number of 'dots' in the region defined by $0 < x < 2$ and $y^2 - 4y + z^2 + 6z \leq -12$ is bigger than 100,

(b) the distance from the origin to its 3^{th} nearest dot is between 25 and 35 cm (centimeter is one hundredth of a meter).