

Full credit given for 3 (out of 5) correct and complete answers.

Give all answers to at least 4 significant digits.

All final results **must** be entered in your booklet.

Your Maple is to be e-mailed to [jvrbik@brock.ca](mailto:jvrbik@brock.ca)

Open-book exam.

Duration: 1 hour

1. Assume that customers arrive at the following rate

$$15 + \frac{9t - t^2}{5} \quad \text{per hour}$$

where  $t$  is time from the store's opening at 9:00, in hours (e.g.  $t = 2$  means 11:00). The store closes at 5 pm (the arrival rate then drops down to 0). Compute

- (a) the probability of fewer than 13 arrivals between 11:37 and 12:14,
  - (b) the expected time of the arrival of the 5<sup>th</sup> customer after opening (use the xx:yy:zz format) and the corresponding standard deviation (in minutes and seconds).
2. Suppose that store's customers arrive at the rate of 7.8 clusters per hour, where the size of each cluster has the following distribution

cluster size:	1	2	3	4
Pr:	0.37	0.41	0.18	0.04

Compute the probability that, during the next 17 minutes, the store welcomes

- (a) at least 3 clusters of customers,
  - (b) more than 7 customers in total,
  - (c) no clusters of size 4.
3. Consider a  $M/G/\infty$  queue with service times having the uniform distribution between 3 and 7 minutes, and customers arriving at the rate of 8.3 per hour. Find
- (a) the probability that, half an hour after opening (starting with 0 customers), more than 3 customers have already left (having completed their service),
  - (b) the probability that the next idle period (no customer in service) lasts more than 3 minutes,
  - (c) the long-run percentage of time with more than 3 customers being serviced.

4. Customers arrive at a constant rate of 14.7 per hour, but each of them will make a purchase with the probability of only 82% (the rest will only browse). Compute the probability that
- (a) during the next 37 minutes, the store gets at least 8 buying customers and no more than 4 browsers.
  - (b) the store gets its 8<sup>th</sup> buying customer before the 3<sup>rd</sup> browser walks in.
5. Suppose a gas station with an arrival rate of 9.7 customers per hour is open for a random time  $T$ , whose distribution is  $\text{gamma}(5, 46 \text{ minutes})$ . What is the probability of
- (a) getting, during this time, fewer than 59 customers,
  - (b) staying open for more than 6 hours.