MATH 4P85 FIRST MIDTERM OCTOBER 16, 2018

Open book exam. Full credit given for 6 (out of 10) correct (to 4 significant digits) and complete answers. Use of (only!) Maple is permitted.

Duration: 1 hour

1. Visualize a triangular lawn with sides of length 12, 10 and 10 meters. Over this lawn, dandelions grow as a Poisson process with the average density of 3.7 dandelions per square meter. Find

   (a) the probability that next year the lawn will have more than 170 dandelions in total.

   (b) the expected distance from the *middle* of the lawn's longest side to its 3$^{\text{rd}}$ nearest dandelion, and the corresponding standard deviation.

2. Customers enter a store in clusters; these form a Poisson process with the average rate of 12 clusters per hour. The size of each cluster is random, independently selected from the following distribution

# of customers	1	2	3	4
Pr	0.33	0.36	0.23	0.08

   Assuming that we have just started observing the process, find

   (a) the probability that more than 15 *customers* enter the store during the next 42 minutes,

   (b) the probability of seeing more than 10 'small' clusters (of 1 or 2 customers) enter the store before getting 5 'big' ones (of more than 2 customers),

   (c) the expected time of arrival of the 8$^{\text{th}}$ *customer* (from now).

3. Consider an $M/G/\infty$ system which opens at 8:00 with no customers. These then start arriving at the average rate of 11.2 per hour; their individual service times have the $\mathsf{gamma}(3, 2 \text{ min})$ distribution (a sum of 3 independent exponentials, each having the mean of 2 minutes). Find

(a) the probability that the day's 3^{rd} *departure* will happen before 8:25,

(b) the expected time of the 3^{rd} departure (answer using the hr:min:sec format, rounded to the nearest second),

(c) the long-run proportion of time with at least 3 customers in service.

4. Customers enter a store at an average (time-dependent) rate of

$$\lambda(t) = \begin{cases} 2.2 + 0.4t & t < 1 \\ \dfrac{2.6}{1+\sqrt{t}} & t \geq 1 \end{cases}$$

per hour, where t is time (also in hours) set to zero at the store's opening. Find

(a) the probability that the 3^{th} customer will arrive between 50 and 80 minutes after opening,

(b) the expected time (since opening) of the arrival of the 3^{th} customer.