MATH 4P85 FIRST MIDTERM OCTOBER 3

Open book exam. Full credit given for 5 (out of 10) correct answers. Write them all (each to at least 4 significant digits) in your booklet. Maple is allowed, but *not* 'with(Statistics)'; email to jvrbik@brocku.ca

Duration: **50 minutes**

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1. Consider a two-dimensional Poisson process with the average density of 'dots' equal to 1.82 per inch^2. Find the probability of getting more than 17 dots inside the triangle with vertices $(-1, 3)$, $(2, -3)$ and $(4, 0)$; inches are units of our coordinates.

2. Customers arrive at a dollar store at an average rate of 18.2 per hour; each makes a random (from our point of view) purchase whose amount has the *modified* geometric distribution with $p = 0.14$. Find the probability that

   (a) the total amount of money spent by customers who arrive during the next 23 minutes is between \$40 and \$60 (inclusive),

   (b) the store will get at least 7 'big spenders' (those who spend *more* than \$10 each) during the next two and a half hours.

3. Consider an $M/G/\infty$ queue with the average arrival rate of 17.3 customers per *hour*, and service times having a distribution with the following probability density function

$$g(s) = \begin{cases} \dfrac{s-5}{24} & 5 < s \le 8 \\ \dfrac{1}{8} & 8 < s \le 13 \\ \dfrac{16-s}{24} & 13 < s \le 16 \\ 0 & \text{otherwise} \end{cases}$$

   where s is the service time in *minutes*. The service opens at 8:00 (with no customers waiting at the door). Find

   (a) the long-run proportion of time with more than 4 busy (i.e. attending to an customer) servers,

(b) the probability that, by 8:32, more than 3 customers have already finished their service.

4. The average rate of customer arrivals varies with time and is given by the following function

$$\lambda(t) = \frac{15t}{9 + t^2} \qquad \text{per hour}$$

where t is time *since* the store's opening at 8:00, in hours; thus, for example, 9:47 corresponds to $t = 1 + \frac{47}{60}$ etc. Find the probability that the 4^{th} customer of the day arrives between 8:45 and 9:30.

5. Consider a Poisson process with the constant arrival rate of 12.8 per *hour*, observed for a time interval whose duration is random and has the gamma(5, 3 minutes) distribution. Compute

   (a) the expected number of arrivals observed during that time, and the corresponding standard deviation,

   (b) the probability that, during that time, there are fewer than 6 arrivals.

6. Consider the following random experiment: Roll two dice followed by flipping a coin as many times as the total number of dots shown on the two dice. Compute

   (a) the expected number of *heads* thus obtained, and the corresponding standard deviation,

   (b) the probability of getting fewer than 5 heads.